

STATISTICS (Paper II)

Time Allowed : Three Hours
Maximum Marks : 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION 'A'

1. (a) Explain the term 'control' in connection with Statistical Quality Control. Distinguish between Process control and Product control. 10
- (b) Define (i) Type I censoring, (ii) Type II censoring and (iii) Random censoring. Describe the situations in which they may arise, either by design or due to experimental circumstances. 10
- (c) In the context of a rectangular game, give the concepts of (i) payoff matrix, (ii) pure strategy of players and (iii) mixed strategy of players. 10
- (d) For a $(M/M/1):(\infty|FCFS)$ queue system, compute– (i) expected number of customers in the system; (ii) expected queue length; (iii) expected (average) waiting time of a customer in the system. 10
- (e) Mention the importance of sensitivity analysis. What are the different problems that are resolved through it? 10
2. (a) State the single sampling plan for attributes. Explain Dodge-Romig system of determination of plan parameters for (i) LTPD plans and (ii) AOQL plans. 15
- (b) What do you mean by “process is statistically controlled”? How to detect lack of control using a control chart? State the control limits for fraction defective control chart when population proportion is unknown and sample sizes are (i) fixed and (ii) variable. 15
- (c) Define (i) the reliability function and (ii) the failure rate function of a random variable denoting lifetime of a component. Establish the relation between them, if any. Prove that if $h(t)$ is the failure rate function, then $\int_0^\infty h(t) dt = \infty$. 20
3. (a) Explain the meanings of (i) basic solutions and (ii) feasible solutions in a linear programming problem with m conditions and n variables. Using simplex method, solve the following linear programming problem:

$$\begin{aligned} & \text{[Maximize } Z = 3x_1 + 2x_2 + 5x_3 \text{ ; subject to } x_1 + 2x_2 + x_3 \leq 430 \text{ ; } 3x_1 + 2x_3 \leq 460 \text{ ;} \\ & \quad 3x_1 + 4x_2 \leq 420 \text{ ; } x_1, x_2, x_3 \geq 0] \end{aligned}$$

15
- (b) Discuss Economic Lot Size model with constant demand and variable order cycle time. 15

- (c) Consider a single-server queue system with Poisson input and exponential service time. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible number of calling units in the system is two. Derive the steady-state probability distribution of the number of calling units in the system and also calculate the expected number in the system. 20

4. (a) Let there be three components with independently and exponentially distributed lifetimes each with mean 1000 hours. Find the mean lifetime of (i) the series system and (ii) the parallel system of these components. 20

- (b) What do you understand by an assignment problem? Mentioning the different steps of solving an assignment problem, find the solution of the following assignment problem:

Employees \ Tasks	T_1	T_2	T_3	T_4	T_5
E_1	25	55	60	45	30
E_2	45	65	55	35	40
E_3	10	35	45	55	65
E_4	40	30	70	40	60
E_5	55	45	40	55	10

15

- (c) What are the different costs involved in an inventory model? Obtain the formulae for optimal order quantity and optimum variable inventory cost for the inventory model with constant demand and instantaneous supply. 15

SECTION 'B'

5. (a) Explain the difference between Paasche's and Laspeyres' index numbers. Check whether they satisfy the time reversal and factor reversal tests. 10

- (b) What are the objectives of analyzing seasonal movement in a time series? How does it differ from the cyclical movement? Describe the method of simple averages to deseasonalize a time series. 10

- (c) Explain different (i) methods of collecting census data and (ii) types of error in census data. 10

- (d) Discuss registration of vital statistics in India, stating its uses and limitations. 10

- (e) Explain clearly the term 'scaling' as used in the problems of scholastic achievement. How does this scale differ from ordinary metre scale? 10

6. (a) Name any two central statistical organizations in India and explain their functions. 20

- (b) Explain the need of weighted least squares estimators over OLS estimators. Describe the models in which this is required. 15

- (c) India's imports (in '000 crores) for 1980–81 to 1986–87 are given below:

Year	1980–81	1981–82	1982–83	1983–84	1984–85	1985–86	1986–87
Imports	125.5	136.1	142.9	158.3	171.3	197.7	200.8

Fit a linear trend by the method of least squares, and plot the observed values and trend values in the same graph paper. 15

7. (a) Compute Age Specific Fertility Rate (ASFR) and Total Fertility Rate (TFR) from the following table:

Age group (in years)	Female population ('000)	No. of live births
15–19	200.6	4227
20–24	173.5	26099
25–29	161.7	32844
30–34	160.9	23449
35–39	155.7	11588
40–44	125.6	2071
45–49	87.6	122

15

- (b) Explain the method of population projection using logistic curve. State its limitations. 15
- (c) Obtain the general Spearman-Brown formula and explain how it is used for estimating reliability by the split-half method. What is the effect of increasing the length of a perfectly reliable test on reliability? 20

8. (a) The following table gives the number of female births classified by age of mothers and survival rates of mothers:

Age of mothers (in years)	Female population ('00)	No. of female live births ('00)	Survival rate (per 100000)
15–19	157670	4632	58065
20–24	147624	14443	55870
25–29	124200	14058	52981
30–34	105865	8329	48963
35–39	89264	4036	44146
40–44	77887	2158	39154
45–49	61161	689	34198

Compute Gross Reproduction Rate (GRR) and Net Reproduction Rate (NRR), and draw your conclusion. 20

- (b) What is the difficulty in ranking individuals on the basis of total of raw scores on different tests? How to overcome this difficulty? Explain linear derived scores in this connection. 15
- (c) The following data relate to the wholesale prices of six different cereals in January and June of a year:

Cereals	Weight	Price in January (in ₹)	Price in June (in ₹)
1	3.68	240	290
2	2.25	174	161
3	0.42	135	165
4	0.18	128	173
5	0.05	133	105
6	0.19	147	180

Calculate the index number for June with January as the base by weighted aggregative method. 15