

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION 'A'

1. (a) Describe a CUSUM control chart. Compare this chart with a Shewhart chart with respect to performance. How is a V-mask useful in CUSUM charts? 10
 - (b) Let $\{X_n, n=0,1,2,\dots\}$ be a Markov chain on the state space $\{a,b,c\}$ with initial probability values $q_a=2/5, q_b=1/5, q_c=2/5$ and one-step transition probability matrix $((p_{ij}))$ given by

$$([0 \ 1/3 \ 2/3; 1/4 \ 3/4 \ 0; 2/3 \ 0 \ 1/3])$$

Compute (i) $P_r[X_1=b, X_2=b, X_3=b, X_4=a, X_5=c | X_0=a]$. (ii) $P_r[X_1=b, X_2=b, X_3=a]$. 10
 - (c) Discuss the estimation of the parameter of exponential distribution in case of time censoring. 10
 - (d) To a single server queuing system, the arrivals of customers are according to a Poisson process at the rate of 2 customers per hour. The service time has exponential distribution with mean service time of 12 minutes per customer. The steady state probability distribution (p_n) for the number of customers (n) is given below:
- | | | | | | | | |
|-------|------|-----|-----|-----|-----|------|-------------|
| n | 0 | 1 | 2 | 3 | 4 | 5 | 6 and above |
| p_n | 0.08 | 0.1 | 0.4 | 0.2 | 0.2 | 0.02 | 0 |
- Compute (i) the expected number of customers in the system and in the queue. (ii) the mean waiting time in the system and in the queue. 10
- (e) Describe a two-person zero-sum game and explain mini-max, max-min and net values of a game. 10
2. (a) Explain the concept of sequential sampling plan for attributes and obtain the average sampling number function of a sequential sampling plan for testing the hypothesis $H_0: \beta = \beta_0$ against the alternative hypothesis $H_1: \beta = \beta_1$ in sampling from a population with density function $f(x, \beta)$. 15
 - (b) State and prove the necessary and sufficient condition for the existence of a saddle point in a two-person zero-sum game. Find the value of the game (rows: Player A; columns: Player B)

$$[[9 \ 3 \ 1 \ 8 \ 0; 6 \ 5 \ 4 \ 6 \ 7; 2 \ 4 \ 3 \ 3 \ 8; 5 \ 6 \ 2 \ 2 \ 1]]$$
 20
 - (c) Define and explain the following terms: (i) Reliability function (ii) Reliability of a series system (iii) Reliability of a parallel system. The mean life of a component is 100 hours. If you want to build a parallel system having a mean life of 200 hours, how many components would be required considering a constant hazard model? 15

3. (a) For an $(M/M/1):(\infty/\text{FIFO})$ queuing system, derive the set of steady-state difference equations and hence find the solution of the system. In a supermarket, the average arrival rate of customers is 5 every 30 minutes. The average time it takes to list and calculate the customers purchases at the cash desk is 4.5 minutes and this time is exponentially distributed. (i) How long will the customers expect to wait for service at the cash desk? (ii) What is the probability that the cashier is working? 15

- (b) Describe the principles on which a control chart is based. What is the concept behind the use of $3\text{-}\sigma$ limits in statistical quality control? Control on measurements of pitch diameter of threads in aircraft fittings is checked with 5 successive items measured at regular intervals. 8 such samples are given below: (Values are expressed in units of 0.0001 inch)

Sample	Measurement on each item of 5 items per hour				
1	46	45	44	43	42
2	41	41	45	42	41
3	40	40	43	42	40
4	42	43	43	42	45
5	42	44	47	47	45
6	39	46	45	42	41
7	46	44	41	40	45
8	40	45	43	40	39

Construct the $\bar{X}$ -R chart and comment on the process control. 20

- (c) Discuss the inventory model with instantaneous stochastic demand with continuous replenishment unit and obtain the condition for finding the optimum inventory level. 15

4. (a) What do you understand by the term “Sensitivity Analysis”? Discuss the effect of variation in the resource vector b in the linear programming problem with set of constraints $AX=b$. In the problem

Maximize $Z = -x_1 + 2x_2 - x_3$ subject to $3x_1 + x_2 - x_3 \leq 10$, $-x_1 + 4x_2 + x_3 \geq 6$, $x_2 + x_3 \leq 4$, $x_1, x_2, x_3 \geq 0$
discuss the effect of discrete change in the component b_2 so as to maintain the optimality of the current optimum solution. 20

- (b) Explain the terms (i) Acceptance quality level (ii) LTPD (iii) Consumers’ risk (iv) Producers’ risk (v) OC curve 10

- (c) Suppose that the purchaser and the ender agree that a defective rate of 10% in a lot is satisfactory. For the sampling plan $(N=20, n=5, c=1)$, determine the probability that a lot with this per cent (i.e. 10%) defective, will be accepted. Also determine the said probability if the sampling plan happens to be $N=100, n=5, c=1$. Which of these two sampling plans seems better and why? 20

SECTION 'B'

5. (a) Let $\epsilon_t, \epsilon_{t+1}, \dots$ and ξ be independent variables with zero mean and unit variance. Considering $U_t = a\xi + \epsilon_t$, $-\infty < t < \infty$, show that the process is stationary with correlogram $\rho_1 = \rho_2 = \dots = a^2/(1+a^2)$. 10

- (b) What is validity of Test Scores? How you will calculate validity? Also make comparison with the concept of reliability. 10

- (c) Define index numbers. Discuss their uses and limitations. 10

- (d) Describe a method of fitting trend by logistic curve. 10

- (e) Why the gross reproduction rate (GRR) and net reproduction rate (NRR) are considered as refined measures of fertility? Assuming that the ratio of female babies to total birth is 48.8 per cent, compute the gross reproduction rate for the following data:

Age group	16-20	21-25	26-30	31-35	36-40	41-45	46-50
-----------	-------	-------	-------	-------	-------	-------	-------

10

Fertility rate per thousand women	19	173	253	201	157	67	9
-----------------------------------	----	-----	-----	-----	-----	----	---

6. (a) Fill in the blanks in a portion of the life table given below:

Age in years	l_x	d_x	q_x	P_x	L_x	T_x	E_x
4	95000	500	?	?	?	4850300	?
5	?	400	?	?	?	?	?

20

- (b) Following are two sets of indices one with 1960 as base and the other as 1970 as base:

Year	1960	1961	1966	1968	1970	1971	1972	1973
Index (Old)	100	110	115	125	150	...	...	...
Index (New)	...	...	...	...	100	105	120	130

Splice the new series to old series, so as to have a continuous series from 1960. Also splice the old series to new series so as to have a continuous series from 1960.

15

- (c) Given below is the data regarding deaths in two districts. On the basis of the given data, calculate the standardised death rates. Give your comments.

Age Range	Population (0 0) District A	Population (0 0) District B	Number of deaths (0 0) District A	Number of deaths (0 0) District B	Age Distribution of Standard 1000
0-10	2000	1000	50	20	206
10-55	7000	3000	75	30	583
55 and above	1000	2000	25	40	211

15

7. (a) For estimating simultaneous linear system of equations, how would you use the following methods of estimation and why? (i) Indirect least squares (ILS) method. (ii) Two-stage least squares (2 SLS) method.

20

- (b) What is meant by intelligence quotient (I.Q.)? Describe the procedure and tests for measuring it.

15

- (c) Elucidate ARIMA (p,d,q) representation of a time series. For an autoregressive scheme $X_{t+1} = aX_t + b + \epsilon_{t+1}$ show that the correlogram is given by $\gamma_k = a^k$, where k is the order of the serial correlation. Also, show that the correlogram is decreasing exponentially for $0 < a < 1$.

15

8. (a) Explain the time reversal and factor reversal tests of index numbers. Examine whether Laspeyre's and Paasche's index numbers satisfy these tests.

15

- (b) What do you mean by stable and quasi-stable populations? Stating the basic assumptions of stable population theory, derive three basic equations which provide information about the intrinsic growth rate, birth rate and age distribution of the population.

20

- (c) Explain the terms (i) summation score as a function of ability (ii) percentile scores (iii) Z-score. Describe how T-scores are to be found when we are given test scores in the form of a frequency table.

15