

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. (a) Describe single and double sampling plans used in acceptance sampling. Define the operating characteristic function of a sampling plan. 12
- (b) The cost of maintenance of a machine is given as an increasing function of time and its scrap value is constant. If time is measured continuously, then show that the average cost will be minimized by replacing the machine when the average cost to date becomes equal to the current maintenance cost. 12
- (c) Describe how the control chart based on sample ranges is constructed. 12
- (d) Describe Type I and Type II censoring. Discuss the consequence of such censoring in the analysis of lifetime data. 12
- (e) Show that in a linear programming problem, the feasible solutions form a convex set. 12
2. (a) A person needs at least 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product available in the market contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. The price of the liquid product is ₹ 3 per jar and that of the dry product is ₹ 2 per carton. How many jars and cartons should be purchased to meet the requirements and to minimize the cost? 30
- (b) Consider a zero-sum game between two persons A and B, where each player has three strategies A_1, A_2, A_3 and B_1, B_2, B_3 . Player A's payoff matrix is given below:

Player A \ Player B	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7
A_3	-7	4	0

Is there a saddle point? What are the optimal strategies for the two players? 15
- (c) Arrivals at a telephone booth are considered to be according to a Poisson process with an average time of 10 minutes between two consecutive arrivals. The length of a phone call is assumed to be exponentially distributed with a mean of 3 minutes. What is the probability that a person coming to the booth will have to wait? What is the average length of the queues that form from time to time? 15
3. (a) A sample of 100 screws was selected on each of 25 successive days in a factory manufacturing screws, and each screw was examined for defects. The data for number of unacceptable screws on different days are given below:

Day	No. of defective screws	Day	No. of defective screws
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1	7	14	6
2	4	15	2
3	3	16	9
4	6	17	7
5	4	18	6
6	9	19	7
7	6	20	11
8	7	21	6
9	5	22	7
10	3	23	4
11	7	24	8
12	8	25	6
13	4		

Assume that the production process was in control during the period. Determine the upper and the lower control limits (UCL and LCL) based on this data for the proportion of defective items.

- (b) Consider an acceptance sampling plan where 50 items sampled from a huge lot will be examined and the lot will be accepted if at most two of the sampled items are found defective – otherwise the lot will be rejected. Evaluate the acceptance probability as a function of the proportion of defectives in the lot and sketch the Operating Characteristic (OC) curve. (Only a rough sketch is required and no graph paper is necessary)

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4. (a) The following table gives the maintenance cost per year and the resale price in ₹ of a certain machine whose purchase price is ₹ 5,000:

Year	Maintenance cost	Resale price
1	1500	3500
2	1600	2500
3	1800	1700
4	2100	1200
5	2500	800
6	2900	500

In which year is the replacement due?

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- (b) Consider a Markov chain with three states 0, 1 and 2. The state 0 is an absorbing state, while from any of the other two states the chain moves into any of the three states with equal probability (i.e. $p(a|b)=1/3$ for $a=0,1,2$ and $b=1,2$). Starting from either state 1 or 2, let N = the number of steps or transitions needed before absorption into state 0 takes place. Derive the probability distribution of N . What is the expected number of steps or transitions before absorption into state 0?

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SECTION 'B'

5. (a) For the AR(1) process defined as $X_t = 0.6 X_{t-1} + \varepsilon_t$, where $t=1,2,3,\dots$, $X_0=0$ and $\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots$ are independent $N(0, \sigma^2)$ variables, what is the expression for the spectrum (i.e., spectral density)?
- (b) Describe the problem of multi-collinearity in multiple linear regression. How does it affect parameter estimation? Describe some solution for this problem.
- (c) Describe Laspeyres' and Paasche's methods for computing price indices. Discuss how Marshall – Edgeworth formula leads to a compromise between the two methods.
- (d) Describe the exponential growth curve. Explain how one checks whether a given time series has exponential growth pattern and how one can fit an exponential trend to a time series.
- (e) What is the percentile score of a student in a test? Suppose that the original scores of the student in a test are approximately normally distributed. Then what will be the mean and the variance of the percentile scores?

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6. (a) Consider the linear model
- $$y_{ij} = \alpha_i + \beta_j + \varepsilon_{ij}, \quad i=1, \dots, I, \quad j=1, \dots, J,$$
- ε_{ij} 's are independent $N(0, \sigma^2)$ variables. Obtain the least squares estimates of $\alpha_r - \alpha_s$ and $\beta_p - \beta_q$ for $r, s=1, \dots, I, r \neq s$ and $p, q=1, \dots, J, p \neq q$. Obtain also the variance expressions for those least squares estimates. 30
- (b) Derive the normal equations for fitting a linear trend to time series data. Suppose that a time series X_t has been observed at time points $t=\pm 1, \pm 3, \pm 5, \dots, \pm 19, \pm 21$ and for this series $\sum_t X_t = 528.2$ and $\sum_t tX_t = -1956.4$. Determine the linear trend equation that will fit the data. 30
7. (a) What is the Logistic curve? Describe in detail the method of Pearl and Reed and also Rhodes' method for fitting a logistic curve to population data. 30
- (b) Define Irving Fisher's "ideal" price index. Explain why it is called an "ideal" index by giving relevant mathematical details. Give an example of well-known price-index formula that is not "ideal" and explain why it is so. 30
8. (a) Give a brief outline of factor analysis and discuss its importance in Psychometric studies. 15
- (b) Distinguish between an abridged life table and a complete life table. Discuss a method of constructing abridged life table. 15
- (c) What is reliability of test scores and how is it determined? 15
- (d) Write a note on official agencies responsible for data collection on trade and prices. 15