

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer the following:

12×5=60

- (a) For a finite irreducible aperiodic Markov chain with n -step transition probabilities $P_{ij}^{(n)}$, show that $\lim_{n \rightarrow \infty} P_{ij}^{(n)}$ exists and is independent of the initial state.
- (b) Describe briefly about the queue system (M/G/1). Obtain the expected number of persons in the system from $P(z)$.
- (c) State the procedure for double sampling attributes plan for lot disposition. Also derive its OC and ASN functions.
- (d) Use simplex method to solve the following LPP:

$$\text{Maximize } Z = x_1 - x_2 + 3x_3$$

subject to

$$[x_1 + x_2 + x_3 \leq 10; 2x_1 - x_3 \leq 2; 2x_1 - 2x_2 + 3x_3 \leq 0; x_1, x_2, x_3 \geq 0]$$

- (e) Discuss an inventory model with probabilistic demand.

2. (a) Justify the use of artificial variables in obtaining LPP. When should slack and surplus variables be used for solving linear programming problem? **10**

(b) Explain the economic cost project models used in queueing theory. **20**

(c) State the limitations for game theory. Following is the pay-off matrix for players A and B. Using dominance property, obtain the optimum strategies for both the players and determine the value of the game:

Player A \ Player B	I	II	III	IV	V
I	2	4	3	8	4
II	5	6	3	7	8
III	6	7	9	8	7
IV	4	2	8	4	3

10

(d) Explain the concept of recurrence and transience in the case of finite Markov chains. **20**

3. (a) Determine the optimal strategies for the game with the following pay-off matrix:

A \ B	I	II	III
I	4	3	1
II	0	1	2

25

- (b) A supermarket has 2 girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the rate of 10 per hour– (i) what is the probability of having to wait for service; (ii) what is the expected percentage of idle time for each girl? 20

- (c) A company uses annually 24000 units of a raw material which costs ₹ 1.25 per unit. Placing each order costs ₹ 22.5 and the carrying cost is 5.4% per year of the average inventory. Find the economic lot-size and the total inventory cost (including cost of material). 15

4. (a) The following table shows all the necessary information on the available supply to each warehouse, the requirements for each market and the unit transportation cost from each warehouse to each market:

Warehouse \ Market	I	II	III	IV	Supply
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
Requirement	7	12	17	9	

The shipping clerk has worked out the following schedule from experience: 12 units from A to II; 1 unit from A to III; 9 units from A to IV; 15 units from B to III; 1 unit from C to III; 7 units from C to I. (i) Check and verify if the clerk has the optimal schedule. (ii) Find the optimal schedule and the minimum total shipping cost. 25

- (b) Construct a control chart for $\bar{X}$ and R from the following data on the basis of samples of 5 being taken every hour (each set of 5 has been arranged in ascending order of magnitude). Comment on the state of control. What will be the future control limits for $\bar{X}$ and R charts?

42	42	19	36	42	51	60	18	15	69	64	61
65	45	24	54	51	74	60	20	30	109	90	78
75	68	80	69	57	75	72	27	39	117	93	94
78	72	81	77	59	78	95	42	54	118	109	109
87	90	81	84	78	132	135	60	62	153	112	139

Table Value: For $n=5$: $A_2=0.577$, $D_3=0$, $D_4=2.11$ 20

- (c) Explain the different types of redundancies and their role in improving the reliability of a system. 15

SECTION 'B'

5. Answer the following:

12×5=60

- (a) Explain the identification problem, and state the rank and order conditions for identifying a model equation. Show that these conditions are necessary and sufficient.
- (b) Define and describe quasi-stable theory of population.
- (c) State Laspeyre's and Paasche's index number of prices, and examine whether they satisfy the various tests for index numbers.
- (d) Explain the concept for moving average model and autoregressive moving average model [ARMA (p,q)].
- (e) Explain how the infant mortality rate is constructed.

6. (a) Explain Durbin-Watson test for autocorrelation under a general linear model. 20
- (b) Explain in detail two-stage least squares method for estimation. 20
- (c) Describe in detail the problem of multicollinearity with an example. 20
7. (a) Explain in detail the three tests for fixed base index numbers. 25

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| | (b) | Project Weibull distribution as a failure model. Derive its hazard function. What is a bathtub curve? | 20 |
| | (c) | Explain different measures of mortality. Briefly discuss about the procedures for finding standardization of rates through direct and indirect methods. | 15 |
| 8. | (a) | Specify the time and factor reversal tests for a price index number. Illustrate by a standard price index number which satisfies both the tests. Prove your claim. | 25 |
| | (b) | Write a note on Central Statistical Organization. | 20 |
| | (c) | Write briefly on: (i) Methods for collection of official statistics in India (ii) Reliability and limitations of data collection | 15 |