

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Attempt any five of the following:

12×5=60

- (a) If i is a recurrent state with period t with a given mean recurrence time μ_{ii} , then show that

$$\lim_{n \rightarrow \infty} p_{ii}^{nt} = t/\mu_{ii}$$

- (b) In $(M/M/1):(\infty/FCFS)$ queuing model, obtain the expression for the probability that there are n units in the system at any time.

- (c) What is rectifying inspection plan? Obtain the expressions for AOQ and ATI, with reference to a single sampling plan for attributes.

- (d) Solve graphically the following linear programming problem:

$$\text{Min } Z = 3x_1 + 5x_2$$

subject to

$$[-3x_1 + 4x_2 \leq 12; 2x_1 - x_2 \geq -2; 2x_1 + 3x_2 \geq 12; x_1 \leq 4, x_2 \geq 2; x_1, x_2 \geq 0.]$$

- (e) Explain fixed order quantity approach and periodic review approach in Inventory control.

- (f) Explain the replacement policy for item whose running cost increases with time but value of money changes with constant rate during the period.

2. (a) Explain assignment problem and transportation problem. Give its mathematical formulation. **10**

- (b) At time zero, all items in a system are new. Each item has a probability p of failing immediately before the end of the first month of life, and a probability $q=1-p$ of failing immediately before the end of the second month (i.e., all items fail by the end of the second month). If all items are replaced as they fail, then show that the expected number of failures $f(x)$ at the end of month x is given by

$$f(x) = N/(1+q)[-(-q)^{x+1}]$$

where N is the number of items in the system. **20**

- (c) State and prove Fundamental theorem of game theory. **10**

- (d) Suppose there is a product of brands A and B respectively. Let each of these two brands have exactly 50 per cent of the total market in same period and let the market be of a fixed size. The transition matrix is given below:

From \ To	A	B
A	0.9	0.1
B	0.5	0.5

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If the initial market share breakdown is 50% for each brand, then determine their market shares in the steady-state.

3. (a) For any zero-sum two-persons game where the optimal strategies are not pure strategies and for which A's pay-off matrix is

A \ B	y_1 I	y_2 II
x_1 I	a_{11}	a_{12}
x_2 II	a_{21}	a_{22}

Show that the optimal strategies are (x_1, x_2) and (y_1, y_2) given by $x_1/x_2 = (a_{22} - a_{21}) / (a_{11} - a_{12})$ and $y_1/y_2 = (a_{22} - a_{12}) / (a_{11} - a_{21})$ and the value of the game to A is given by $v = (a_{11}a_{22} - a_{12}a_{21}) / ((a_{11} + a_{22}) - (a_{12} + a_{21}))$.

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- (b) A repairman is to be hired to repair machines that breakdown following a Poisson process with an average rate of four per hour. The cost of non-productive machine is Rs. 90 per hour. The company has the option of choosing either a fast repairman who charges Rs. 60 per hour and repairs machines at the rate of 7 per hour or a slow repairman who charges Rs. 30 per hour and repairs machines at the rate of 5 per hour. Taking the cost in to the consideration find which repairman should be hired.

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- (c) An aircraft company uses rivets at a constant rate of 2500 per year. Each unit costs Rs. 30 and the company personnel estimate that it costs Rs. 130 to place an order, and that the carrying cost of inventory is 10% per year. How frequently should orders be placed? Also determine the optimum size of each order.

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4. (a) A firm manufacturing a single product has three plants X, Y and Z. The three plants have produced 60, 35 and 40 units respectively during this month. The firm had made commitment to sell 22 units to customer A, 45 units to customer B, 20 units to customer C, 18 units to customer D and 30 units to customer E. Determine the minimum possible transportation cost of shifting the manufactured products to the five customers. The net per unit cost of transporting from the three plants to five customers is given below:

Plant \ Customer	A	B	C	D	E
X	4	1	3	4	4
Y	2	3	2	2	3
Z	3	5	2	4	4

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- (b) The following data (pertaining to two sub-groups of size 4), is from two different machines which are supposed to be alike. Plot the necessary charts to show whether their product would support this assumption. If they do not support this assumption, does this prove the machines are not essentially alike?

Subgroup No.	Machine-I Average	Machine-I Range	Machine-II Average	Machine-II Range
1	2.77	0.06	2.53	0.12
2	2.7	0.29	2.67	0.3
3	2.78	0.19	2.66	0.17
4	2.67	0.12	2.57	0.25
5	2.75	0.34	2.6	0.24
6	2.77	0.23	2.6	0.05
7	2.75	0.17	2.7	0.3
8	2.73	0.06	2.56	0.04
9	2.76	0.23	2.7	0.19
10	2.63	0.2	2.67	0.08
11	2.73	0.17	2.6	0.11
12	2.73	0.28	2.63	0.14
13	2.74	0.26	2.71	0.24
14	2.72	0.13	2.63	0.31
15	2.73	0.13	2.75	0.17

Table values:

N	3	4	5
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d_2	1.693	2.059	2.526
A_2	1.02	0.73	0.58

- (c) Define Reliability. List out some of the problems in achieving reliability. How you will distinguish between Reliability and quality control? 15

SECTION 'B'

5. Answer any five of the following:

12×5=60

- (a) What is identification problem? Explain the rank and order conditions of identifiability.
- (b) Define Time series. Explain briefly different components of Time series.
- (c) Explain Quasi Stable theory of Population.
- (d) Explain the concept of Auto covariance, Auto correlation and their functions. Also explain how the functions are estimated.
- (e) Define Index Number. What does an Index Number mean? Explain the uses of Index Numbers.
- (f) Explain briefly the concept of path analysis. Mention their uses in Psychometry.
6. (a) Explain the concept of Validity. How one can estimate the Validity? Explain what are the different types of validity? Also discuss the concept of test length with respect to validity. 20
- (b) What are the four important stages in ARIMA modelling of a times series data? Explain briefly. 20
- (c) What is Heteroscedasticity? Write its nature and consequences. Describe the Spearman's rank correlation test useful for detecting Heteroscedasticity. 20
7. (a) Show that Fisher's ideal index number and Marsall Edgeworth index number lie between Laspeyre's and Paasche's index numbers. 25
- (b) A given test has a reliability coefficient of 0.8 and standard deviation of 20. (i) What is the maximum correlation which this test is capable of yielding as it stands? (ii) What is the S.E. of a score obtained on this test. (iii) What is the estimated reliability coefficient of this test in a group in which standard deviation is 15? (iv) What proportion of the variance of the scores in this test is attributable to "true" variance? 20
- (c) Explain different measures of mortality. Briefly discuss about the procedure for finding standardization of rates through direct and indirect methods. 15
8. (a) What is Cost of living Index? Mention the main steps in construction of the cost of living index values. How you will construct it? Explain both the methods. 25
- (b) Explain briefly the system of collection of industrial statistics in India. Also mention its reliability and limitations. 20
- (c) With usual notation show that (i) $e_x = ((\sum_{n=1}^{\infty} l_{x+n})/l_x)$ (ii) $e_x^0 = T_x/l_x$ (iii) $p_x = e_x/(1+e_{x+1})$ 15