

STATISTICS (Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Attempt any five sub-parts:
 - (a) Let X be an absolutely continuous random variable with p.d.f. $f(x)$ and distribution function $F(x)$. Show that $Y=F(x)$ has $U(0,1)$ distribution. 12
 - (b) Let $X_1, X_2, \dots$ be i.i.d. random variables with common p.d.f.

$$f(x) = \begin{cases} [(1+\delta)/(x^{2+\delta})], & x \geq 1 \ (\delta > 0) \\ 0, & \text{otherwise} \end{cases}$$
 Show that the weak law of large numbers holds for the sequence. 12
 - (c) Let X be a random variable with p.m.f. $P(X=-2)=1/5$, $P(X=-1)=1/6$, $P(X=0)=1/5$, $P(X=1)=1/15$, $P(X=2)=11/30$. Obtain the distribution of $Y=X^2$. 12
 - (d) Let T_1 and T_2 be two unbiased estimators of θ having the same variance. Show that their correlation coefficient ρ cannot be smaller than $2e_\theta - 1$, where e_θ is the efficiency of each estimator. 12
 - (e) If a sufficient statistic T exists for the family $\{f_\theta; \theta \in \Theta\} = \Theta = \{\theta_0, \theta_1\}$, show that the Neyman-Pearson MP test is a function of T . 12
 - (f) Outline the Wilcoxon-Mann-Whitney test for the two-sample problem. Also, give the relevant large-sample test-statistic. 12
2. (a) If X is any non-negative integer valued variate and 'a' is any positive number, show that $P[X \geq a] \leq t^{-a} E[t^X]$, $t > 1$ and verify the inequality $P[X \geq 2\lambda] \leq (e/4)^\lambda$ when X follows Poisson distribution with parameter. 20
- (b) Let F_n be a sequence of distribution functions defined by

$$F_n(x) = \begin{cases} 0, & x < 0 \\ 1 - 1/n, & 0 \leq x < n \\ 1, & x \geq n \end{cases}$$
 Does X_n converge in distribution to X ? If so, identify the distribution function of X . Also, examine whether $E[X_n^k] \rightarrow E[X^k]$ for any k as $n \rightarrow \infty$. 20
- (c) The characteristic function of a random variable X is $(q + pe^{it})^n$. Find the distribution of X . 20
3. (a) Let $X_1, X_2, \dots, X_n$ be a sample from p.d.f.

$$f(x, \theta) = \begin{cases} [x/\theta e^{-x^2/2\theta}], & x > 0, \theta > 0 \\ 0, & x \leq 0 \end{cases}$$
 Show that $\sum_{i=1}^n X_i^2$ is a minimal sufficient statistic for θ , but $\sum_{i=1}^n X_i$ is not sufficient. 20
- (b) Let $X_1, X_2, \dots, X_n$ be a random sample from

$$f(x, \theta) = \begin{cases} [\beta x^{\beta-1}], & 0 < x < 1, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$
 Find the MP test of size α for testing $H_0: \beta = 1$ against $H_1: \beta = 2$. Also obtain its power. 20

- (c) Let the power function of every test of $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$ [?] be continuous in θ . Show that a UMP α -similar test is UMPU provided that its size is α for testing H_0 against H_1 20
4. (a) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from a distribution $b(1, \theta)$, $0 < \theta < 1$. Obtain the UMVU estimator of $(\theta(1-\theta))/n$ 20
- (b) State and prove a result which shows that, compared to any other sequential test procedure, the SPRT has the least ASN. 20
- (c) For each $\theta_0 \in \Theta$, let $A(\theta_0)$ be the acceptance region of the level- α UMP test for $H_0:\theta=\theta_0$ against all alternatives. Show that $S_0(x)$ defined by $S_0(x) = \{\theta | x \in A_0(\theta)\}$ is, for varying $x \in X$ [?] a family of UMA confidence sets at the level $1-\alpha$. 20

SECTION 'B'

5. Attempt any five sub-parts
- (a) If X (with p components) is distributed according to $N_p(\mu, \Sigma)$, then prove that $Z=DX$ is distributed according to $N_q(D\mu, D\Sigma D')$ where D is a $q \times p$ matrix of rank $q \leq p$. 12
- (b) Show that a linear function of observations belongs to the error space if and only if its coefficient vector is orthogonal to the columns of the matrix X in the usual linear model $Y=X\beta+\epsilon$. 12
- (c) A simple random sample of size $n=n_1+n_2$ with mean $\bar{y}$ is drawn from a finite population, and a simple random sub-sample of size n_1 is drawn from it with mean $\bar{y}_1$. Show that (i) $V(\bar{y}_1 - \bar{y}) = (1/n_1 - 1/n)S^2$. (ii) $Cov(\bar{y}_1 - \bar{y}, \bar{y}_1 - \bar{y}) = 0$ 12
- (d) Bring out, giving an example, the relationship between a sampling scheme and a sampling design. 12
- (e) Find a confounding arrangement of a 2^5 factorial experiment in 4 blocks such that no main effect or first order interaction is confounded. 12
- (f) What is the purpose of MANOVA? Explain the one-way MANOVA model. 12
6. (a) For the model
- $$y_1 = \beta_1 + \beta_2 + \epsilon_1, y_2 = \beta_1 + \beta_3 + \epsilon_2, y_3 = \beta_1 + \beta_2 + \epsilon_3$$
- show that $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable if and only if $\lambda_1 = \lambda_2 + \lambda_3$. 20
- (b) Explain the purpose of discriminant analysis. What is Fisher's discriminant function for two populations? Give the allocation rule based on it. 20
- (c) Outlining the purpose of factor analysis, describe the orthogonal factor model along with its covariance structure. 20
7. (a)(i) Describe a method of selecting a sample with probability proportional to size and with replacement. 10
- (a)(ii) Obtain the variance and its estimator for the Hansen-Hurwitz estimator. 10
- (b) Show that the usual regression estimator is unbiased under a suitable model to be specified by you. Also, obtain the model variance of this estimator. 20
- (c) Using Warner's randomized response technique, obtain the estimate of the population proportion relating to the sensitive characteristic. 20
8. (a) Give the analysis of an LSD if the yield of one of the plots is missing. 20
- (b) Derive the analysis of variance table for 2-way classification with m (> 1) observations per cell in the case of a random effects model. 20
- (c) Construct a confounded 2^6 factorial experiment into 8 blocks of 8 treatments each by confounding the effects ABEF, ABCD and ACE. Also, identify the other effects which 20

are confounded.