

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Attempt any FIVE sub parts of the following:

- (a) What are trial control limits? What is done if a point is found outside trial control limits? Explain why periodic revision of control limits and central line of a control chart is necessary. 12
- (b) What is the V-mask procedure? Is it useful for a one-sided process monitoring problem? If not, why? 12
- (c) Derive the OC function of a single sampling inspection plan. Define AQL and LTPD corresponding to a lot. Explain how to choose a plan with specified values of AQL and LWD. 12
- (d) Define the primal LP problem and its dual. Show that the optimal values of objective function under both the primal and dual problems are equal, if they are finite 12
- (e) Define pure and mixed strategies in a two person zero sum game. The following is the payoff matrix for player A for such a game between A and B.

	B_1	B_2	B_3
A_1	5.0	50.0	50.0
A_2	1.0	1.0	0.1
A_3	10.0	1.0	10.0

- (f) Show that the mixed strategies $(1/6, 0, 5/6)$ and $(49/54, 5/54, 0)$ are respectively optimal for A and B. 12
- (f) Consider the following transition probability matrix of a Markov chain

$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{bmatrix}$$
 Classify the states of the associated MC. 12

2. (a) Define (i) a series system, (ii) a parallel system. Let there be three independent components with reliabilities 0.9, 0.8 and 0.7 respectively. What are the reliabilities of the series system, and of the parallel system composed of these components? 20
- (b) A parallel system is formed of two independent components with exponential life distributions. Show that the hazard rate of the system is not constant. 15
- (c) Explain how are the $\bar{X}$ and the R charts maintained. Discuss why it is important to maintain both of these simultaneously. 25

3. (a) Show that the set of all feasible solutions of a L.P. problem is a convex set, bounded by hyperplanes. 15
- (b) Describe the transportation problem. Explain what is meant by (i) the north-west corner method, and (ii) the least cost method of initializing the solution. What are the subsequent steps in each of these methods? 20
- (c) Explain all the terms in the notation $(M/M/1):(FCFS/\infty/\infty)$, λ , μ ; in the context of queues. Let p_n be the steady state probability of n customers in the above queuing system. Show that $p_n = (1-\rho)\rho^n$, where $\rho = \lambda/\mu$. 25
4. (a) Let $N(t)$, the number of times a system fails upto time t , be a Poisson process with $E(N(t)) = \mu t$. Show that the system does not fail in an interval of length Δ with probability $e^{-\mu\Delta}$ 15
- (b) Explain the concepts of Type I, Type II and Random censoring. Let T , the lifetime of a unit have exponential distribution with mean $1/\mu$. Find the maximum likelihood estimate of μ and a 95% confidence interval for it in case the data is subject to Type II censoring. 15
- (c) Apply Vogel approximation method for solving the transportation problem with 3 sources and 4 destinations with the following table of unit transportation costs: 15
- | Source \ Destination | 1 | 2 | 3 | 4 | Supply |
|----------------------|----|----|----|----|--------|
| 1 | 10 | 2 | 20 | 11 | 15 |
| 2 | 12 | 7 | 9 | 20 | 25 |
| 3 | 4 | 14 | 16 | 18 | 10 |
| Demand | 5 | 15 | 15 | 15 | |
- (d) Describe the salient features of the SPSS package. Explain how to find a multiple linear regression based on p independent variables and n ($\geq p$) observations. 15

SECTION 'B'

5. Attempt any five subparts of the following:
- (a) Define a second order stationary time series. What does the autocorrelation coefficient ρ_k of such a series represent? 12
- (b) Explain how is the base year of a series of index numbers selected. Why is it necessary to shift the base year from time to time? 12
- (c) What is the nodal government agency for the coordination of all statistical data collection in India? What are its major functions and how are they organized? 12
- (d) Explain the concept of force of mortality μ_x at age x . Express it in terms of l_x and its derivative. Can μ_x be decomposed into several cause specific forces (in a multiple decrement life table)? 12
- (e) What are the principal publications which provide data on: (i) Foreign exchange transactions (ii) Banks and Non-banking financial corporations (iii) Joint stock companies (iv) Mining activity 12
- (f) Define 'intelligence quotient'. What are the salient features of the 'Stanford-Binet' test? Is 'Finding Reasons' a component of the test at all ages? 12
6. (a) What are the components in which a time series is usually decomposed? Explain their roles in understanding the behaviour and predicting future values of a time series. Give one example each where these components are expected to be (i) additive, (ii) multiplicative, justifying these structures in your examples. 20
- (b) How does one detect heteroscedasticity in linear – models? What are the implications of heteroscedasticity in estimating the regression coefficients? 20

- (c) In the standard linear model with $Y=X\beta+\epsilon$, with $\epsilon \sim N(0, \sigma^2)$, show that the maximum likelihood estimates $\hat{\beta}$ of β have the distribution $\hat{\beta} \sim N(\beta, \sigma^2(X'X)^{-1})$ assuming $X'X$ to be an invertible matrix. 20

7. (a) Explain the terms l_x, T_x, e_x, p_x, d_x and L_x appearing in a life table. How are these related to ${}_nq_x, {}_np_x, {}_nd_x$ and ${}_nL_x$ appearing in an abridged life table? Is such modification required for l_x, T_x and e_x also? 20

- (b) What are the features of a standard population? Calculate the crude and standardized death rates using the data in the following table:

Age group	Population ('000)	No. of deaths	Age distribution in standard population
0 - 9	21	350	221
10 - 24	30	102	298
25 - 44	37	227	285
45 - 64	17	354	149
65 +	5	415 [?]	47
			1000

- (c) Explain why crude birth rate does not measure the fertility of a population. Describe general fertility rate, age specific fertility rate and total fertility rate. Why is the total fertility rate independent of the age distribution in the population? 20

8. (a) What are the different heads of origin which account for the net domestic product of India? Which agency coordinates the calculation of the net domestic product? 20

- (b) What are standard scores? Why are they used instead of raw scores for further analysis? Describe and explain the difference between Z-scores and T-scores. 20

- (c) Explain the role of the Reserve Bank of India in collection and publication of economic/financial data and indices. 20

- (d) What are the aspects of human behaviour to which psychological tests correspond? Discuss how one checks whether a test is valid for measuring these aspects. 20