Civil Services (Main) Examination, 2003

STATISTICS

(Paper I)

Time Allowed : Three Hours **Maximum Marks : 300**

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.
Candidate has to attempt FIVE questions in all.
Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.
The number of marks carried by a question/part is indicated against it.
Wherever any assumptions are made for answering a question, they must be clearly indicated.
Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any FIVE sub-parts

(a) For the three mutually exclusive events A, B, C, the following probabilities are given $P(A)=2/3$, $P(B)=1/4$, $P(C)=1/6$. Calculate $P(A \quad B \quad C)$ and comment about result. **12**

(b) Define characteristic function $\phi(t)$. State its properties. Examine whether the following relations are characteristic function: (i) $\phi(t)=(3/5e^{it}+1).e^{5(e^{it}-1)}.e^{5it-t^2}$ (ii) $\phi(t)=e^{4it+t^2}$ **12**

(c) State the properties of the probability density function of a random variable X. Examine whether the following f(x) is a probability density function. Also evaluate $P(2<x<6)$:

$$f(x)=\{[1/\theta e^{-x/\theta}, \quad 0<x<\infty ; 0, \quad \text{elsewhere}]$$

Where $\theta>0$ is a constant. **12**

(d) Let X be a normal variate with mean μ and variance σ^2 Estimate μ and σ^2 by using method of moments. **12**

(e) Explaining the notations describe Chi-square good-ness of fit test. **12**

(f) Suppose X_1 and X_2 are identically and identically distributed Poisson variates with parameter θ. Show that the statistic $T=X_1+3X_2$ is not sufficient for θ. **12**

2. (a) State the conditions for the existence of weak law of large numbers. Examine whether weak law of large numbers holds for the mean of a sequence of independent variates X with $P\{X_k=\pm\sqrt{(\log k)}\}=1/2$ **20**

(b) The joint probability density function of two random variables X and Y is $f(x,y)=1/8(6-x-y)$, $0<x<2$, $2<y<4$ Find (i) $P(X<1,Y<3)$ and $P(X+Y<3)$ (ii) Marginal distributions of X and Y. **20**

(c) Let X be a random variable having standard Cauchy's distribution. Obtain the probability density function of X^2 and identify the resulting distribution. **20**

3. (a) Explain the concept of consistency. Show that for Cauchy's distribution the sample median is a consistent estimator of the population mean. **20**

(b) State Chapman-Robbins inequality. Explain how it differs from Cramer-Rao inequality **20**

(c) Prove that if $\hat{\theta}$ is the maximum likelihood estimator of θ and $\phi(\theta)$ is a one-to-one function of θ, then $\phi(\hat{\theta})$ is the maximum likelihood estimator for $\phi(\theta)$. **20**

4. (a) A sample of size 1 is taken from probability density function, $f(x:\theta)=(2(\theta-x))/\theta^2$, $0<x<\theta$ Find most powerful test of $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$, $\theta_0>\theta_1$ [?], at level α. **20**

(b) Discuss Wilcoxon-Mann-Whitney test and examine its consistency. **20**

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(c) Define likelihood ratio test and discuss its consistency. **20**

SECTION 'B'

5. Answer any FIVE sub-parts:

(a) If $Y=X\beta+e$ is a general linear hypothesis model of full rank. Find the estimate of β the unknown parameter using appropriate method when the distribution of e is unspecified **12**

(b) Define regression analysis. Write the form of simple linear regression model. State the various assumptions of the model. **12**

(c) Define regression estimator. When it becomes consistent? Write the reasons for becoming regression estimator biased. **12**

(d) Define cluster sampling. Write the basic difference between the cluster sampling and stratified sampling. **12**

(e) If D denotes a BIBD with parameters (v,b,r,k,λ) and D^* denotes its complementary design with parameters $(v^*,b^*,r^*,k^*,\lambda^*)$, then show that, $\lambda^*=b-2r+\lambda$. **12**

(f) Define total and partial confounding. In the following arrangement of 2^3 factorial designs, identify the confounded factorial effects:-

Replicate I, Block I	Replicate I, Block II	Replicate II, Block I	Replicate II, Block II	Replicate III, Block I	Replicate III, Block II
abc	ab	abc	ac	abc	ab
a	ac	ab	bc	bc	ac
b	bc	c	a	a	b
c	'1'	'1'	b	'1'	c

12

6. (a) Let X be distributed $N(\mu,\Sigma)$, find the characteristic function of X. **20**

(b) What are the situations where discriminant analysis is used? Give some examples in support of your answer. Write the objectives of two group discriminant analysis. Explain how the discriminant criterion is calculated. **20**

(c) Explain the concept of one-way classified data. Write the linear model for it Obtain the least square estimate of the parameters involved in it. **20**

7. (a) If finite population correction is ignored, then show that $V_{opt}\leq V_{prop}\leq V_{srs}$ where V_{opt}, V_{prop} and V_{srs} denote variance of optimum allocation, allocation, proportional allocation and simple random sample respectively. **20**

(b) Prove that the variance of the mean $\bar{y}_{sy}$ of systematic sample is

$$V(\bar{y}_{sy})=(nk-1)/nk.S^2/n\{1+(n-1)\rho\}.$$

where $nk=N$ and ρ is the intraclass correlation coefficient between the units of the same systematic sample. **20**

(c) Explain the concept of ratio estimator. Prove that in large sample, with simple random sampling, the ratio estimate $\hat{Y}_R$ has a smaller variance than the population estimate $\hat{Y}$ obtained by simple expansion, if

$$\rho>(\text{coefficient of variation of } x_i)/(2(\text{coefficient of variation of } y_i))$$ **20**

8. (a) Define symmetrical factorial experiment. Obtain all possible main effects and intersections for 2^4 factorial experiment. Discuss Yates method of analysis of 2^n factorial experiment. **20**

(b) Give the layout of a randomized block design considering 6 treatments into 5 blocks. Whether this is a balanced design? Give the complete analysis of variance of randomized block design with k treatments and r blocks. (Derivation of formulae not needed). **20**

(c) Define simple lattice design. Construct a simple lattice design taking 16 treatments. Derive the complete analysis of simple lattice design. **20**

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