

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Attempt any five sub-parts:

- (a) Distinguish between process control and product control. Discuss the advantages of process control. 12
- (b) Describe the procedure for setting up control chart for proportion defective for varying sample size 12
- (c) Explain the concepts of (i) producers risk, (ii) consumer's risk. (iii) average outgoing quality limit (AOQL) and (iv) average total inspection (ATI). Give significance of AOQL and ATI in acceptance sampling plans. 12
- (d) What is a linear programming problem? What are the assumptions made in a linear programming problem? The manager of an oil refinery company must decide on the optimal mix of two possible blending processes of which the inputs and the outputs per production run are as follows:

Processes	Input (units): Crude-1	Input (units): Crude-2	Output (units): Petrol superior	Output (units): Petrol ordinary
A	10	6	10	16
B	12	15	12	12

- The availability of the two varieties is limited to the extent of 400 units and 450 units, respectively, per day. The market demand indicates that at least 200 units and 240 units of superior and ordinary quality petrol, respectively, is required every day. The profitability analysis indicates that the process A contributed Rs. 480 per run while the process B contributed Rs. 400 per run. The manager is interested in determining an optimal product mix to maximize company's profit. Formulate this as a linear programming problem. 12
- (e) What is a two-person zero-sum game? Distinguish between pure and mixed strategies. Solve the following game:

Player – A \ Player – B	B_1	B_2
A_1	3	5
A_2	7	4

- (f) The following is the transition probability matrix of a Markov chain:

P	1	2	3
1	0	1	0
2	1/2	0	1/2
3	0	1	0

- Examine whether the Markov chain is regular and ergodic. 12

2. (a) Explain the following terms: (i) specification limits, (ii) tolerance limits, (iii) control limits and (iv) warning limits. A machine is to manufacture plugs to a nominal diameter of 0.400 ± 0.030 inches. Ten samples, each of size 5, were taken from online production and the following statistics were obtained:

Sample number	Sample mean	Sample range
1	0.4952	0.017
2	0.4962	0.016
3	0.5032	0.028
4	0.5014	0.023
5	0.5021	0.018
6	0.5018	0.038
7	0.4960	0.011
8	0.4952	0.012
9	0.5036	0.020
10	0.4988	0.013

Set up control charts for mean and range and comment if the manufacturing process meets the specifications. Calculate warning limits for mean. (For $n=5$, $D_2=2.326$, $D_3=0$, $D_4=2.115$)

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- (b) Derive single sampling inspection plan for variables when the upper specification limit is specified and the operating characteristic curve passes through two fixed points: (AQL, $1-\alpha$) and (LTPD, β). You may assume that the lot quality characteristic is normally distributed and the standard deviation is known.

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- (c) Consider a two-component parallel system with a single repairman available for maintenance. Suppose that the failure and repair processes are both Markovian with rates a and b , respectively, Show that the system reliability is

$$R(t) = ((3a+b+s_1)e^{s_1 t} - (3a+b+s_2)e^{s_2 t}) / ((s_1 - s_2))$$

where $(s-s_1)(s-s_2) = s^2 + (3a+b)s + 2a^2$ and the Laplace transform of $p(t)$ is $P(s)$.

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3. (a) Write down the dual of the following linear programming problem and solve it by using simplex method

Minimize $Y = 24y_1 + 21y_2 + 9y_3$ subject to $y_1 + 3y_2 + y_3 \geq 2$, $4y_1 + y_2 + y_3 \geq 5$, $y_1 \geq 0$, $y_2 \geq 0$, $y_3 \geq 0$.

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- (b) Define a transportation problem. Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method. Also, obtain the optimal solution by MODI method:

Origin \ Destination	D_1	D_2	D_3	D_4	Supply
O_1	10	2	20	11	15
O_2	12	7	9	20	25
O_3	4	14	16	18	10
Demand	5	15	15	15	50

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- (c) What are the elements of queueing theory? For the queueing model M/M/1 : FCFS/ ∞ , derive the steady state solution. The arrivals at a counter in a bank occur in accordance with Poisson law at an average rate of 8 persons per hour. The duration of service of a customer has exponential distribution with a mean of 6 minutes. Find the probability that a customer (i) has to wait on arrival, (ii) finds 4 customers in the queue and (iii) has to spend less than 15 minutes in the bank. Estimate also the fraction of the total time for which the customer is busy. (It is given that $e^{-0.5} = 0.6065$).

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4. (a) Explain the concept of sequential sampling inspection plan for attributes, In testing $H_0: p=p_0$ and $H_1: p=p_1$ (where p is proportion of defective items in a lot), determine acceptance and rejection lines.

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- (b) The failure time distribution of a component is given by: $f_x(x) = \theta e^{-\theta x}$; $x > 0$, $\theta > 0$ Show that the hazard rate is θ for all $x > 0$. If the exponential distribution is truncated from above at $x=k$, then show that the survival distribution function is not exponential but if the truncation is

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made at $x=k$ from below, it is an exponential distribution.

- (c) Discuss briefly sensitivity analysis in the context of a linear programming problem: Maximize $Z=C'X$ subject to $AX\leq b$, $X\geq 0$, pertaining to instantaneous changes in (i) the coefficients of the objective function, (ii) b and (iii) coefficients a_{ij} in the matrix A . 15
- (d) Explain the use of SPSS package in solving statistical problems. In particular, how will you carry out two-way analysis of variance for a given data by using SPSS? 15

SECTION 'B'

5. Attempt any five sub-parts
- (a) Explain how the trend in a time series is determined by the method of moving averages 12
- (b) What is an index number? Describe how the index number for wholesale prices and the cost of living index number are constructed. 12
- (c) Describe briefly the present official statistical system in India relating to (i) population statistics and (ii) agricultural statistics. 12
- (d) What do you mean by population projection? Discuss the component method of population projection. 12
- (e) Write a critical note on health surveys and use of hospital statistics in India. 12
- (f) What do you mean by intelligence quotient? How is it measured? What are its uses? 12
6. (a) What are the different methods of determining cyclical component of a time series? For the model $u_t = a \sin 2\pi t / \lambda + e_t$, give the periodogram analysis. 20
- (b) Explain the term autocorrelation. What are the consequences of auto-correlation? Explain how the Durbin-Watson d statistic is used to detect the presence of autocorrelation? 20
- (c) Discuss the problem of heteroscedasticity in linear models. Describe Glejser test for detecting heteroscedasticity. What are the difficulties in using Glejser test? How do you overcome these difficulties? 20
7. (a) Explain how the crude birth rate, general fertility rate, age-specific fertility rate and total fertility rate are measured. 15
- (b) What are abridged life tables? How does an abridged life table differ from a complete life table? Explain Greville's method of constructing abridged life table. 25
- (c) Explain Rhodes method of fitting a logistic curve to the census population data of a country. 20
8. (a) What are the principal publications that provide data on: (i) agricultural statistics, (ii) industrial statistics and (iii) trade statistics? 15
- (b) What are the various official agencies responsible for data collection? Discuss their main functions? 15
- (c) Explain the term 'validity'. Also, explain briefly the four types of validity concepts. 15
- (d) Describe the uses of factor analysis and path analysis in Psychometry. 15