

STATISTICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any five sub-parts:
 - (a) If $P(A)=p$ and $P(B)=q$, then show that $P(A|B) \geq (p+q-1)/q$ 12
 - (b) Let $\{X_n\}$ be a sequence of random variables with p.m.f. $P(X_n=1)=1/n$, $P(X_n=0)=1-1/n$. Show that $X_n \xrightarrow{P} 0$. 12
 - (c) Show that, for any characteristic function $\phi(t)$, $\text{Re}(1-\phi(t)) \geq 1/4 \text{Re}(1-\phi(2t))$ 12
 - (d) Let X follow a Poisson distribution with parameter λ . Examine whether $t(X)=(-1)^X$ is unbiased for $e^{-2\lambda}$. In any case, comment on the suitability of the proposed estimator. 12
 - (e) Establish the consistency of an estimator under a set of sufficient conditions to be stated by you. 12
 - (f) If (α, β) is the strength of a Sequential Probability Ratio Test with boundary points (A, B) , prove that $A \leq (1-\beta)/\alpha$ and $B \geq \beta/(1-\alpha)$ 12
2.
 - (a) Let $X_1, X_2, \dots, X_n$ be i.i.d. geometric random variables with parameter p . Show that the p.m.f. of the smallest order statistic $X_{(1)}$ is also geometric with parameter $1-(1-p)^n$ 20
 - (b) Let X be a standard normal variate with p.d.f. $f(x)=1/\sqrt{(2\pi)}e^{-x^2/2}$, $-\infty < x < \infty$. Find the characteristic function of X . 20
 - (c) State Liapounov's form of central limit theorem. Let $\{X_k\}$ be a sequence of independent Bernoulli variables with $P(X_k=1)=P_k=1-P(X_k=0)$. Show that $\{X_k\}$ obeys central limit theorem. 20
3.
 - (a) Let X be a binomial variate with parameters n and p ($0 < p < 1$). Using a quadratic error loss function and a priori distribution of p as $\pi(p)=1$. obtain the Bayes' estimate for p . 20
 - (b) Find an MP test of size α for testing $H_0: X \sim 1/\sqrt{(2\pi)}e^{-x^2/2}$, $(-\infty < x < \infty)$ against $H_1: X \sim 1/2 e^{-|x|}$, $(-\infty < x < \infty)$ based on a sample of size 1. 20
 - (c) Discuss Wilcoxon signed rank test, and examine its consistency. 20
4.
 - (a) Let $X_1, X_2, \dots, X_n$ be a random sample from

$$f(x, \theta) = \begin{cases} 1/\theta e^{-x/\theta}, & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$
 Where $0 < \theta < \infty$. Show that $\bar{X} = 1/n \sum_{i=1}^n X_i$ is a minimum variance bound estimator of θ and has variance θ^2/n . 20
 - (b) Stating the regularity conditions, show that the maximum likelihood equation has a solution which is consistent and asymptotically normal. 20

- (c) Let $X_1, X_2, \dots, X_n$ be a random sample from a rectangular distribution with p.d.f.,

$$f(x, \theta) = \begin{cases} 1/\theta & 0 < x < \infty \\ 0, & \text{otherwise} \end{cases}$$
where $0 < \theta < \infty$. Examine if $T = X_{(n)}$, the largest order statistic, is complete. 20

SECTION 'B'

5. Attempt any five sub-parts:
- (a) Let $X' = [X_1, X_2, X_3]$ be $N_3(\mu, \Sigma)$ with $\Sigma = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Are X_1 and X_2 independent? What about independence of (X_1, X_2) and X_3 ? 12
- (b) Establish the range of a multiple correlation coefficient. 12
- (c) Let π_i^* ($i=1, 2, \dots, N$) be the probability of not including the i th unit in the sample and let π_{ij}^* ($i \neq j=1, 2, \dots, N$) be the probability of including neither the i th nor the j th units in the sample. Then, show that $\pi_i^* \pi_j^* - \pi_{ij}^* = \pi_i \pi_j - \pi_{ij}$ where π_i is the probability of including the i th unit in the sample and π_{ij} is the probability of including both the i th and the j th units in the sample. 12
- (d) If $\bar{y}_R$ is the ratio estimator of the population mean $\bar{Y}$, then prove that $|\text{Bias}(\bar{y}_R)|/SE(\bar{y}_R) \leq CV(\bar{x})$ where $CV(\bar{x})$ is the coefficient of variation of the sample mean $\bar{x}$. 12
- (e) Define, giving examples, the terms 'main effects' and 'two factor interactions'. 12
- (f) Define Mahalanobis D^2 . Show that it is invariant under any non-singular linear transformation. 12
6. (a) Explain the linear regression model. Calculate the least squares estimate of $\beta = [\beta_0; \beta_1]$ for a straight line model $Y_j = \beta_0 + \beta_1 Z_{j1} + \epsilon_j$ from the data
- | | | | | | |
|-------|---|---|---|---|---|
| Z_j | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 4 | 3 | 8 | 9 |
- 20
- (b) If X and Y are two independent p -dimensional random variables and if $X + Y$ has a p -variate normal distribution, then prove that both X and Y have p -variate normal distributions. 20
- (c) What are principal components? Discuss the method of finding them. 20
7. (a) Describe Lahiri's method of selecting a sample with probability proportional to size and with replacement. For this method, show that the average number of draws to select one unit is $M/\bar{X}$ where M is the maximum of sizes X_i ($i=1, 2, \dots, N$) and $\bar{X} = 1/N \sum_{i=1}^N X_i$. 20
- (b) Describe any two methods of improving the performance of systematic sampling in the presence of a linear trend. 20
- (c) Obtain the necessary form of a non-negative unbiased estimator of the variance of the Horvitz-Thompson estimator. 20
8. (a) Give, in detail, the analysis of 2^3 -factorial experiment conducted in r randomized blocks. 20
- (b) Explain the missing plot technique. In an experiment arranged in the form of a randomized block design, the yield of a single plot is ruined by pests. Indicate, giving theory, as to how you will analyze the experiment. 20
- (c) Explain the structure of a split-plot design, giving a practical situation where it can be used. Derive the expected value of mean square due to whole-plot treatments. 20