

STATISTICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Attempt any five sub-parts:
 - (a) Explain how you will detect lack of statistical control in a manufacturing process using a control chart. 12
 - (b) Given p_1 , $1-\alpha$, p_2 and β (usual notations), obtain the item by item sequential sampling plan for attributes. 12
 - (c) Define the terms reliability, maintainability and availability. Also obtain the reliability of an item whose life time distribution is given by $f(t)=\alpha\alpha t^{\alpha-1}e^{-\alpha t^\alpha}$; $\alpha\alpha < 0$ 12
 - (d) Show that the inter-occurrence time follows the exponential distribution in the case of a Poisson process. 12
 - (e) What do you understand by sensitivity analysis with reference to a Linear Programming Problem? Explain. 12
 - (f) Explain the graphical method of solving a rectangular game. 12
2. (a) Explain the terms AOQ and ATI. For a double Sampling attributes plan with rectifying inspection derive the expressions for the AOQ and the ATI. 20
 - (b) A system consists of n components and it will function if and only if at least k ($1 \leq k \leq n$) of the n components function. If the life time of the i^{th} component is exponential with parameter λ_i find the reliability of the system. 12
 - (c) Distinguish between the p- and the np-charts. Also explain the construction and operation of the latter. 15
3. (a) Obtain the rules for choosing the incoming variable and the outgoing variable in the simplex method of solving a linear programming problem. 25
 - (b) For a finite irreducible aperiodic Markov chain with n -step transition probabilities $p_{ij}^{(n)}$ show that $\lim_{n \rightarrow \infty} p_{ij}^{(n)}$ exists and is independent of the initial state. 20
 - (c) A baking company sells its cakes by weight. It makes a profit of Rs. 0.95 for every kg (kilogram) of cake sold on the day it is baked. It disposes off all cakes not sold on the day they are baked at a loss of Rs. 1.15 per kg. If the demand for the cake is known to have the probability density $f(d)=0.03-0.0000d$, $0 \leq d \leq c$, find the optimum amount of cake the company should bake daily. 15
4. (a) What are $\bar{X}$ and R charts? When are they used? Explain how they are constructed and analyzed. Also explain how you would estimate the process standards using these charts. 15

- (b) Write a brief note on MIL-STD-414 tables. 15
- (c) Define the dual of a linear programming problem (LPP). Also explain the relationships between the solutions of the primal LPP and its dual. If the primal problem has an unbounded solution what can you say about the solution of the dual? 15
- (d) Show that every zero-sum two person game problem can be formulated as a LPP. 15

SECTION 'B'

5. Attempt any five sub-parts:
- (a) Explain Box-Jenkins methodology in analyzing a time series. 12
- (b) State Laspeyres' and Paasche's index numbers for prices and examine whether they satisfy the various tests for index numbers. 12
- (c) What organizations are responsible for collecting population statistics in India? Discuss the types of data collected by them. 12
- (d) A population grows in such a manner that $1/P(t)dP(t)/dt$ is a linear function of $\log P(t)$ where $P(t)$ is the population size at time t . Obtain the explicit expression for $P(t)$ and state its properties. 12
- (e) Explain the problems in developing rates for morbidity. Also define crude and specific morbidity prevalence rates. 12
- (f) What is intelligence quotient? How is it constructed? 12
6. (a) What is a stationary time series? How do you test the stationary of a given time series? 20
- (b) Explain the generalized least squares (GLS) method of estimation. Also obtain the GLS estimate of β in the model $y_t = \beta x_t + u_t$, $t=1,2,\dots,n$ with $E(u_t)=0$, $E(u_t^2)=\sigma^2 x_t$ and $E(u_t u_{t'})=0$ if $t \neq t'$ 25
- (c) Explain the identification problem and state the rank and order conditions for indenting a model equation. Are these conditions necessary and sufficient? 15
7. (a) What is a complete life table? Explain how such a table is constructed. 25
- (b) Explain how the infant mortality rate is constructed. 10
- (c) What are stable and quasi-stable populations? Derive Lotka's equation in this context and explain its utility. 25
8. (a) What are the agencies engaged in the collection and publication of price statistics in India? Also discuss the type of statistics published, their reliability and limitations. 20
- (b) Explain the functions of any two of the organizations engaged in the collection of statistics of industrial production, both under the organized and the unorganized sectors. Where are these statistics published? 10
- (c) Explain the utility of σ -scores and standard scores in analyzing psychological data. State the assumptions under which they are obtained. Under what conditions will these scores be equal? 15
- (d) Explain factor analysis and its uses in psychological investigations. 15