

## STATISTICS (Paper II)

**Time Allowed : Three Hours**

**Maximum Marks : 300**

### QUESTION PAPER SPECIFIC INSTRUCTIONS

*(Please read each of the following instructions carefully before attempting questions)*

There are TWENTY questions divided in five Sections.

Candidate has to select any THREE Sections and attempt FIVE questions in all from them.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

### SECTION 'A'

1. (a) Suppose one university is interested in determining the attitude of students towards a proposed change in the method of examination in its affiliated colleges and teaching departments. How would you proceed to plan a sample survey for this purpose. 20
- (b) Suppose  $N_1$  of the  $N$  units in a population are known to have the value zero and a simple random sample of  $n$  units is selected from the  $N - N_1$  non-zero units with replacement. Suggest an unbiased estimator of the overall population mean and obtain its efficiency compared to simple random sampling of  $n$  units directly from all the  $N$  units with replacement. 20
- (c) Suppose a stratified sampling design is adopted for estimating the overall proportion  $P$  of units having a specified characteristic by dividing the population of units into  $K$  strata and selecting  $n$  units from  $N_i$  units in the  $i^{th}$  stratum ( $i=1,2,\dots,k$ ), with equal probability and without replacement. Suggest an unbiased estimator of  $P$  and obtain its sampling variance. 20
2. (a) Explain the ratio method of estimation. Derive the conditions, if any, under which a ratio estimator of the population mean  $\bar{Y}$  using the information on an auxiliary variable  $X$ , is more efficient than the sample mean  $\bar{y}$  in the case of simple random sampling without replacement, stating clearly the assumptions involved. 20
- (b) Indicating what is a regression estimator, state under what circumstances you would use this estimator in preference to a ratio estimator. 20
- (c) Explain clearly the concept of PPS sampling. What are its advantages and disadvantages? 20
3. (a) Explain uniformity trials. Discuss their use in picking up a proper design. 15
- (b) Explain Fisher's missing plot technique and indicate how you will carry out the exact analysis of variance for a randomized block design with one missing observation. 30
- (c) Write a note on fractional replication. 15
4. (a) Comment on the necessity, advantages and disadvantages of factorial experiments and explain the "odd and even rule" for writing down the expressions for main effects and interactions in  $2^n$  factorial experiments. 20
- (b) Give an outline of the analysis of balanced incomplete block design and obtain its efficiency compared to a randomized block design. 25
- (c) Write an explanatory note on split-plot experiments. 15

### SECTION 'B'

5. (a) Describe the importance of normal distribution in the theory of quality control. 15

|    |     |                                                                                                                                                                                                       |    |
|----|-----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
|    | (b) | Distinguish between 'Process Control' and 'Product Control' Examine the basis of the p-chart and explain the approximations and assumptions involved and how they are met in actual plant conditions. | 30 |
|    | (c) | Distinguish between chance and assignable causes of variation.                                                                                                                                        | 15 |
| 6. | (a) | Explain single and double sampling inspection plans for attributes. What are the statistical advantages of double sampling plan?                                                                      | 15 |
|    | (b) | Given lot size (N), LTPD ( $p_L$ ), consumer's risk ( $P_c$ ) and process average ( $\bar{p}$ ) [?], derive the most economical single sampling plan for acceptance purposes.                         | 30 |
|    | (c) | Define AOQL and describe how to obtain the same for the double sampling plan.                                                                                                                         | 15 |
| 7. | (a) | Write a note on sequential sampling inspection plans for attributes.                                                                                                                                  | 20 |
|    | (b) | Discuss the role of censored samples in life testing experiments.                                                                                                                                     | 20 |
|    | (c) | Define the terms hazard rate and reliability function with their important properties. Obtain them in the case when the life-time distribution is assumed one-parameter exponential.                  | 20 |
| 8. | (a) | Discuss the estimation of reliability characteristics in Weibull distribution with censored sample.                                                                                                   | 20 |
|    | (b) | Obtain the MLE and UMVUE of the parameter of a one-parameter exponential distribution when first r failure times are recorded. Which of the two will you prefer?                                      | 20 |
|    | (c) | What do you understand by hot and cold use of redundancy in reliability improvement? Discuss.                                                                                                         | 20 |

### SECTION 'C'

|     |     |                                                                                                                                                                                                            |    |
|-----|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 9.  | (a) | Explain the concept, scope and tools of OR as applicable to business and industry.                                                                                                                         | 20 |
|     | (b) | What are the basic steps in constructing a model? Describe how to decide whether a model could optimize profits, sales, costs or other trends. Illustrate your answer with suitable examples.              | 20 |
|     | (c) | What do you understand by Markov chains? Explain how it can be used for predicting sales-force needs.                                                                                                      | 20 |
| 10. | (a) | Discuss $(M/M/1):(\infty/FCFS)$ queueing model and find the expected line length in the system.                                                                                                            | 20 |
|     | (b) | State the general linear programming problem. Give its mathematical formulation and explain the terms: (i) Basic solution (ii) Feasible solution (iii) Basic feasible solution, and (iv) Optimal solution. | 20 |
|     | (c) | Show that the optimal solution of a linear programming problem can be found amongst its feasible solutions.                                                                                                | 20 |
| 11. | (a) | Discuss An inventors' model with probabilistic demand.                                                                                                                                                     | 15 |
|     | (b) | Define assignment problem and give its mathematical formulation. Also give an algorithm to solve an assignment problem.                                                                                    | 25 |
|     | (c) | Explain any one method for solving a transportation problem. Would you recommend this method to solve an assignment problem?                                                                               | 20 |
| 12. | (a) | Describe the inter-relation amongst machine language, FORTRAN compiler, the FORTRAN source program and object code.                                                                                        | 20 |
|     | (b) | Write a note on library functions available in FORTRAN IV.                                                                                                                                                 | 20 |
|     | (c) | Write a FORTRAN program to evaluate the following sum:<br>$S = \sum_{i=1}^{10} ((-1)^i X^{n/2}) / (n(n-1))$                                                                                                | 20 |

### SECTION 'D'

13. (a) Why is the multiplicative model the most commonly used assumption, as compared to additive model, in time series analysis? Discuss the role of the method of moving averages in time series analysis 20
- (b) Explain clearly with the help of an illustration how seasonal index is useful in planning sales or production for specific periods. Are there any limitations of seasonal index? 20
- (c) Elucidate the role of random components in a time series. Discuss the variate difference method in this connection. 20
14. (a) State the properties of an 'ideal' index number and discuss four important index numbers in respect of them. 21
- (b) Explain the meaning and uses of cost of living index number. How are weights assigned to various groups and sub-groups? 20
- (c) Explain what is meant by 'bias' in an index number. Give an example of one index number having an 'upward bias' and one index number having a 'downward bias' 15
15. (a) What do you mean by price elasticity of demand? How will you obtain this measure on the basis of time series data? Discuss the various difficulties that are likely to occur in this connection. 20
- (b) Show that the least squares estimate of the slope coefficient in the simple linear regression equation is inconsistent when both the variables are subject to errors of measurement. 20
- (c) Given the two-equation model:
- $$C_t = \alpha + \beta Y_t + u_t, Y_t = C_t + I_t$$
- where  $C_t$  and  $Y_t$  are endogenous variables and  $I_t$  is an exogenous variable, derive the two-stage least square estimate of  $\beta$  and prove that it is consistent under certain conditions (to be stated). 20
16. (a) Describe how you will test the presence of multicollinearity among the explanatory variables in a linear model. 20
- (b) Explain the concept of identification for a simultaneous equation model. Discuss with an example, how the specification of disturbances can lead to identifying relations in a simultaneous equations model. 20
- (c) Write a note on techniques and models used in demand projections. 20

### SECTION 'E'

17. (a) Explain why the mortality situation of two places cannot usually be compared on the basis of crude death rates. Describe the construction of standardized death rates for this purpose. 20
- (b) What factors govern the growth of population? How do you find out whether the population of a country is increasing or decreasing or remaining stationary? 20
- (c) Write a note on the importance of vital statistics for demographic purposes 20
18. (a) What is meant by force of mortality at age  $x$ ? Derive Makeham's formula starting from suitable assumptions 20
- (b) Explain gross and net reproduction rates. What interpretation will you give if the net reproduction rate is just 1, less than 1 or greater than 1? 20
- (c) Describe the structure of a complete life table. 20
19. (a) Describe a suitable method of constructing an abridged life table. 20
- (b) Describe briefly the component method for population projection mentioning the demographic data which will be required for this purpose. 20

- (c) Give a brief account of the National Sample Survey in the field of demography and show how far the findings have been useful in preparation of Five Year Plans in this respect. **20**
20. (a) Why is it considered desirable to convert raw scores to some standard scores? Define 'standardized scores' and 'normalized scores' and describe how they are derived. **20**
- (b) Explain the term Intelligence Quotient (IQ). Describe any method of measuring it. **20**
- (c) Discuss the various methods to determine the reliability of test scores. **20**