Civil Services (Main) Examination, 1996

STATISTICS
(Paper II)

Time Allowed : Three Hours **Maximum Marks : 300**

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are TWENTY questions divided in five Sections.
Candidate has to select any THREE Sections and attempt FIVE questions in all from them.
All questions carry equal marks.
Wherever any assumptions are made for answering a question, they must be clearly indicated.
Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. (a) Define random sampling. Why are random samples distinctly superior to non-random samples? Describe linear systematic sampling with interval k and show that it is an equal probability sampling method.

(b) State the Horvitz-Thompson estimator of a population total in PPS sampling without replacement. Show that it is unbiased.

(c) State the common reasons for stratifying a population before sampling.

2. (a) Outline the ratio and regression methods of estimating a population total. When and how are these methods useful?

(b) Explain the use of double sampling for stratification.

(c) Write a brief note on “non-sampling errors”.

3. (a) State the basic principles of designed experiments and their role in such experiments.

(b) Give the linear model and outline the method of analysis of a RBD.

(c) Define a BIBD. Show that the parameters of a BIBD satisfy the relations (i) $bk=vr$ (ii) $\lambda(v-1)=r(k-1)$

4. (a) Analyze completely the following data from an experiment which used a Latin Square design.

Order of assembly	Operator I	Operator II	Operator III	Operator IV
1	10 C	14 D	7 A	8 B
2	7 B	18 C	11 D	8 A
3	5 A	10 B	11 C	9 D
4	10 D	10 A	12 B	14 C

The letters A, B, C and D represent four assembly methods. The relevant table value of F for $\alpha=0.05$ is 4.76.

(b) What are factorial experiments? Illustrate the concept of total confounding in such experiments.

(c) Write an explanatory note on simple lattice designs.

SECTION 'B'

5. (a) Explain the significance of the following concepts in the construction of control charts. (i) Assignable causes of quality variation (ii) Rational subgroups

(b) Describe briefly the construction and working of a CUSUM chart for a variable characteristic.

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(c) When and how is a np-chart useful? What is the statistical basis of this chart?

6. (a) Describe the working of a double sampling plan for product control. What are the advantages of this plan as compared to a single sampling plan?

(b) Define the concepts of AOQ and ATI in rectifying inspection. Derive expressions for these under a double sampling plan.

(c) Devise a known sigma variables sampling plan when there is a two-sided specification on the quality characteristic.

7. (a) Define a parallel system. Obtain its reliability function and the mean life of the system if all the units are equally reliable and reliability law is exponential.

(b) Suggest a procedure for estimating the parameters of a two parameter exponential distribution under type II censoring.

(c) Describe a bath-tub failure model and its applications.

8. (a) Explain the use of redundancy in reliability improvement.

(b) Define the terms: (i) Hazard function (ii) IFR and DFR distributions Provide examples.

(c) Illustrate the concepts of (i) availability, and (ii) maintainability.

SECTION 'C'

9. (a) Explain the scope of Operational Research.

(b) Derive the steady-state results (probabilities, mean numbers and mean waiting times) for an M/M/s queue.

(c) What is an imbedded Markov chain? For the M/G/1 queue identify the imbedded Markov chain.

10. (a) Define (i) basic feasible solution, (ii) slack and surplus variables and (iii) artificial variables in the context of a linear programming problem (LPP).

(b) Show that if an optimal solution exists for a LPP then it corresponds to an extreme point of the feasible region.

(c) Describe the simplex algorithm for solving a LPP. How is this procedure modified when some of the variables are unrestricted?

11. (a) Define a transportation problem with m sources and n destinations. Show that in such a problem only $(m+n-1)$ restrictions are linearly independent.

(b) Prove that a transportation problem can never have unbounded solutions.

(c) Outline the Hungarian method of solving an assignment problem.

12. (a) Specify a few replacement policies.

(b) Explain the following control statements in FORTRAN language: (i) Unconditional GO TO (ii) Computed GO TO (iii) Arithmetic IF

(c) Write a FORTRAN program to compute the coefficient of variation from a data set having n ungrouped observations.

SECTION 'D'

13. (a) Specify the additive and multiplicative models for a time series. How are seasonal indices constructed?

(b) State the time and factor reversal tests for an index number. Verify whether the price index numbers due to Paasche and Marshall-Edgeworth satisfy these two tests.

(c) Write a note on variate difference method.

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14. **(a)** Formulate a general linear model (GLM), clearly stating the assumptions. Derive the MLE of the parameter vector when the errors in the GLM follow a multivariate normal distribution.

(b) Explain the procedure for testing the hypothesis $H_0:\beta_1+\beta_2=c$ (some given constant) in the model $y_t=\beta_0+\beta_1 x_{1t}+\beta_2 x_{2t}+u_t$. State the underlying assumptions.

(c) Illustrate a technique which may be used to represent factors like temporal effects and qualitative variables in a GLM.

15. **(a)** Mention the main sources that cause serially correlated disturbances. Outline the Cochrane-Orcutt procedure to estimate the parameters of a linear model when the errors are serially correlated.

(b) Show that measurement error in the explanatory variables poses a serious estimation problem in the set up of a GLM. Outline an estimation method based on instrumental variables in this context.

(c) Explain the multi-collinearity problem. Interpret it geometrically and algebraically. Suggest methods to overcome the problem.

16. **(a)** Compare the ILS and 2 SLS methods of estimation in simultaneous equation models.

(b) Examine for identifiability the two-equation system

$$\beta_{11}y_{1t}+\beta_{12}y_{2t}+\gamma_{11}y_{1t}+\gamma_{12}x_{2t}=u_{1t}$$
$$\beta_{21}y_{1t}+\beta_{22}y_{2t}+\gamma_{21}y_{1t}+\gamma_{22}x_{2t}=u_{2t}$$

(i) as it stands, and (ii) if $\gamma_{12}=0$, $\gamma_{21}=0$.

(c) Write a note on short term economic forecasting.

SECTION 'E'

17. **(a)** Define the following and explain their uses: (i) Density and proximity of population (ii) Infant mortality rate (iii) Net reproduction rate

(b) Stating the underlying assumptions, outline the construction of a life-table.

(c) In the usual notation show

$$q_x=2m_x/(2+m_x)$$

in the setup of a life-table.

18. **(a)** Examine Gompertz curve as a growth model.

(b) Describe stable and quasi-stable population models. How are these models useful for estimating demographic parameters?

(c) Explain the direct method of standardising death rates.

19. **(a)** What is test reliability? How is this measured? Describe one method fully.

(b) Examine the effect of lengthening and repeating a test on test reliability.

(c) A test with 40 items has a reliability coefficient of 0.68. What is the reliability of a comparable test with (i) 80 items, (ii) 100 items?

20. Write explanatory notes on any three of the following:

(a) National sample surveys

(b) IQ tests

(c) Logistic curve

(d) Balancing equation in population studies.

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