

STATISTICS (Paper II)

Time Allowed : Three Hours

Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are TWENTY questions divided in five Sections.

Candidate has to select any THREE Sections and attempt FIVE questions in all from them.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. (a) Define bias, variance and mean squared error of an estimator t for a parameter θ . Obtain a relation [?] among these three quantities. 20
(b) Give one illustration each for use of auxiliary information in sample surveys for (i) selection of sample units, and (ii) construction of an estimator of a population total 20
(c) Define linear and circular systematic sampling. Under linear systematic sampling and with the usual notation, show that
$$V(y_{sy}) = S^2/n((N-1)/N)[1+(n-1)\rho_w]$$
where ρ_w is the correlation coefficient between pairs of units that are in the same systematic sample. 20
2. (a) State the principles for (i) stratifying a finite population, (ii) forming clusters of elements. Explain why these are diametrically opposite. 20
(b) Outline the regression method of estimation. Obtain the large sample variance of the regression estimator of the population mean in simple random sampling. 20
(c) When and how is double sampling useful? 20
3. (a) Explain the role of randomization in designed experiments. Describe an RBD in this context. 20
(b) Obtain an estimate of a missing observation in a Latin Square Design. How does the subsequent analysis differ from the usual case? 20
(c) Define a BIBD and show that (i) $bk=vr$ (ii) $r(k-1)=\lambda(v-1)$ and (iii) $b \geq v$, where v, b, r, k, λ are the parameters of the design. 20
4. (a) What are factorial experiments? Give an illustration. State the advantages of these experiments over one-factor experiments. 20
(b) Explain and illustrate total and partial confounding in factorial experiments. 20
(c) Write a short note on multiple comparison methods. 20

SECTION 'B'

5. (a) Explain and statistical basis of p and c charts. 20
(b) In a manufacturing process it is assessed that the mean has a steady declining trend, though the standard deviation could be taken to be constant. Explain how you would modify the construction of the chart for subgroup means. 20
(c) Explain how a CUSUM chart can be used to detect upward shifts in the process mean. 20

6. (a) Explain the concepts: OC curve, AOQ and consumer's risk. 20
- (b) Derive the equations to the acceptance and rejection lines for a sequential sampling plan by attributes whose OC curve passes through the specified points $(p_1, 1-\alpha)$ and (p_2, β) . 20
- (c) Describe a double sampling plan fully. 20
7. (a) Discuss the relative merits of attributes and variables sampling plans. 20
- (b) Define the terms (i) Life-time distribution (ii) Reliability of a component (iii) Failure rate (iv) IFR and DFR distributions 20
- (c) Examine a Weibull distribution as a failure model. Derive its hazard function. 20
8. (a) In a life testing experiment n identical units are put on test, where the failure time follows an exponential distribution. The life-time of the first r failures is noted. Obtain the ML estimators of (i) mean life-time, and (ii) reliability function 20
- (b) Compare series and parallel systems, each with n units. 20
- (c) Examine the use of redundancy in reliability improvement. 20

SECTION 'C'

9. (a) State the chief characteristics of a queueing process. For a $M|M|1:(FIFO|\infty|\infty)$ queueing system derive the distribution of the waiting time in steady state conditions. 20
- (b) Define recurrent and transient states in a finite Markov chain, if C is a communicating class of states, then show that the states of C are either all recurrent or all transient. 20
- (c) What is an absorbing Markov chain? How is its study useful? 20
10. (a) State the (s, S) policy in an inventory problem. Derive this policy when the demand has a negative exponential distribution. 20
- (b) Describe a multi-period stochastic inventory model allowing for a discount factor. How would you extend this model to the infinite period case? 20
- (c) Write a note on dam type storage models. 20
11. (a) Explain the general formulation of a linear programming problem (LPP). Describe an example. 20
- (b) Define a basic feasible solutions (bfs) and an optimal solution to a LPP. Show that a bfs is an extreme point of the set of feasible solutions and conversely. 20
- (c) A firm manufactures two items A and B. It purchases castings which are then machined, bored and polished. Castings for items A and B cost Rs. 3 and Rs. 4 each and are sold at Rs. 6 and Rs. 7 each, respectively. Running costs of the processing machines are Rs. 20, Rs. 14 and Rs. 17.50 per hour respectively and their hourly capacities are as follows
- | | Item A | Item B |
|-----------|--------|--------|
| Machining | 25 | 40 |
| Boring | 28 | 35 |
| Polishing | 35 | 25 |
- Formulate this problem as a LPP. 20
12. (a) Explain Charne's M-technique for solving a LPP. 20
- (b) Define degeneracy in a LPP. When is this encountered and how is this problem overcome? 20
- (c) Write a note on the duality principle in a LPP. 20

SECTION 'D'

13. (a) Discuss any two important procedures to determine the trend component in a time series. 20

- (b) State Laspeyre's formula for a price index number and examine whether it satisfies time and factor reversal tests. 20
- (c) Describe a procedure for constructing a cost of living index number. 20
14. (a) Specify the general linear model. Show that under certain conditions (to be stated) the least squares and the maximum likelihood methods produce identical estimators of the coefficients in the model. 20
- (b) Explain specification error in a general linear model. What are its consequences? 20
- (c) State the problem of multi-collinearity in a linear model. Give an example. 20
15. (a) State the identification problem. Explain why ordinary least squares cannot be used for estimating equations of a simultaneous system. 20
- (b) Outline the two-stage least squares method, its applicability and properties. 20
- (c) Write a note on short-term forecasting. 20
16. (a) Substantiate or refute the following statements (i) Identification is logically prior to estimation. (ii) Identification problem refers to the structural equation and not the reduced form 20
- (b) Examine for identifiability the system

$$[C = \alpha_0 + \alpha_1 Y_t + \alpha_2 Y_{t-1} + u_t; I_t = \beta_0 + \beta_1 Y_t + \beta_2 Y_{t-1} + v_t; Y_t = C_t + I_t + G_t]$$
if G is the only exogenous variable (apart from the dummy variable for the constant term) and $\beta_1 + \beta_3 = 0$. 20
- (c) Write a note on estimation of demand functions. 20

SECTION 'E'

17. (a) Define the following rates: (i) Infant mortality rate (ii) Crude death rate (iii) Standardised death rate Explain how these are useful in the measurement of mortality. 20
- (b) Explain the terms: (i) A life table (ii) An abridged life table How is an abridged life table constructed? 20
- (c) Describe a logistic growth model. How is this fitted to actual data? 20
18. (a) State and interpret the balancing equation in the study of a human population. Define and explain any two important measures of fertility. 20
- (b) Explain the concept of a stable population. How is this useful in population studies? 20
- (c) Write a note on the use of hospital statistics and their limitations. 20
19. (a) Illustrate the need for scaling of tests. Define (i) a standard score, and (ii) a T-score. 20
- (b) Given a distribution of scores with mean 86 and $\sigma=15$ and the raw scores of Gautham and Gopi as 91 and 83, express these as standard scores in a distribution with a mean of 500 and σ of 100. 20
- (c) Explain and illustrate percentile scaling. 20
20. (a) Describe the following methods for determining the reliability of test scores (i) Test-retest (ii) Alternate or parallel forms 20
- (b) Examine in detail the effect of lengthening or repeating a test upon test reliability. 20
- (c) A test of 50 items has a reliability coefficient of 0.78. What is the reliability coefficient of a test (i) having 100 items comparable to those of the original test? (ii) having 125 items comparable to those of the original test? Comment on the results obtained. 20