

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION 'A'

1. Answer all the following questions:

10×5=50

- (a) Write down precisely the conservation theorems for energy, linear momentum and angular momentum of a particle with their mathematical forms.
- (b) Prove mathematically that the addition of any velocity of a particle to the velocity of light in free space merely reproduces the velocity of light in free space only.
- (c) How does one obtain the angular velocity of the Earth about the North Pole with respect to a fixed star as $7.292 \times 10^{-5} \text{ sec}^{-1}$? Explain your method of calculating the above value.
- (d) A convex lens of focal length 20 cm is placed after a slit of width 0.5 mm. If a plane wave of wavelength 5000 Å falls normally on the slit, calculate the separation between the second minima on either side of the central maximum.
- (e) What is the role of an optical resonator in a laser? Why does one prefer curved mirrors instead of plane mirrors in designing an optical resonator?

2. (a) Find out the angle between the reflected and refracted rays when a parallel beam of light is incident on a dielectric surface at an angle equal to the Brewster's angle. Explain how do you use this concept to produce linearly polarized light. **20**

- (b) Using Poiseuille's formula, show that the volume of a liquid of viscosity coefficient η passing per second through a series of two capillary tubes of lengths l_1 and l_2 having radii r_1 and r_2 is obtained as

$$Q = \pi p / 8\eta [l_1/r_1^4 + l_2/r_2^4],$$

where p is the effective pressure difference across the series. **15**

- (c) Show that the moment of inertia of a circular disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane is $1/2 MR^2$. **15**

3. (a) Draw a neat diagram to explain the scattering of an incident beam of particles by a centre of force. **10**

- (b) Show that the differential scattering cross-section can be expressed as

$$\sigma(\theta) = s/\sin\theta |ds/d\theta|,$$

where s is the impact parameter and θ is the scattering angle. **15**

- (c) Show that the rest mass energy of an electron is 0.51 MeV (use the standard values of the physical parameters). **10**
- (d) Find out the phase and group velocities of a radio wave of frequency $\omega = \sqrt{2} \omega_p$ in the ionosphere (as a dielectric medium) of refractive index $n = \sqrt{1 - \omega_p^2 / \omega^2}$. Here, ω_p is the ionospheric plasma frequency. **15**
4. (a) Define coefficients of viscosity and kinematic viscosity of a fluid. What are Poise and Stokes? **10**
- (b) Write down Poiseuille's formula and mention its limitations in analyzing the flow of a liquid through a capillary tube. **10**
- (c) Using matrix method, find out the equivalent focal length for a combination of two thin lenses of focal lengths f_1 and f_2 separated by a distance a . **10**
- (d) Using the concept of Einstein's A and B coefficients for a two-level atomic system under thermal equilibrium, determine the ratio of the number of atoms per unit volume in the two levels experiencing spontaneous and stimulated emission. How does the principle of population inversion lead to the gain mechanism in the active medium of the laser? **20**

SECTION 'B'

5. Answer all the following questions:

10×5=50

- (a) Under one-dimensional configuration, the charge density is given by $\rho(x) = \rho_0 x/5$; where ρ_0 is a constant charge density. If the electric field $|\mathbf{E}|=0$ at $x=0$ and potential $V=0$ at $x=5$, determine V and $|\mathbf{E}|$.
- (b) A conducting sphere of radius 5 cm has a total charge of 12 nC uniformly distributed on its surface in free space. Determine the displacement vector $\mathbf{D}$ on its surface and outside at a distance r from the centre of the sphere.
- (c) A series RLC circuit has a resistance of 100Ω and an impedance of 210Ω . If this circuit is connected to an a.c. source with an r.m.s. voltage of 220 V, how much is the average power dissipated in the circuit?
- (d) Two spheres A and B having same temperature T are kept in the surroundings of temperature T_0 . Consider $T > T_0$. The spheres are made of same material but have different radii r_A and r_B . Using Stefan – Boltzmann distribution, determine which of these will lose heat by radiation faster.
- (e) A Van der Waals gas undergoes Joule-Kelvin expansion with a pressure drop of 50 atm. If its initial temperature is 300°K, determine its final temperature. (Given Van der Waals constant $a=0.136 \text{ Pa m}^6 \text{ mol}^{-1}$, $b=36.5 \times 10^{-6} \text{ m}^3 \text{ mol}^{-1}$, $C_p=30 \text{ J K}^{-1} \text{ mol}^{-1}$, $R=8.3 \text{ J K}^{-1} \text{ mol}^{-1}$)
6. (a) Derive the equation that represents Poynting's theorem. What is its physical significance? **20**
- (b) A radio station transmits electromagnetic waves isotropically with an average power of 200 kW. Determine the average magnitude of the maximum electric field at a distance of 5 km from it. **15**
- (c) The vapour pressure of an organic substance is $50 \times 10^3 \text{ Pa}$ at 40°C. Its normal boiling point is 80°C. If the substance in vapour phase can be treated like an ideal gas, find the latent heat of vaporization of the substance. **15**
7. (a) A plane electromagnetic wave propagating along $+\hat{z}$ direction is incident normally on the boundary at $z=0$ between medium A ($z<0$) and medium B ($z>0$). Determine the reflection coefficient and transmission coefficient for the wave. **20**

- (b) A series RLC circuit has $R=2\ \Omega$. The energy stored in the circuit decreases by 1% per period of oscillation. Its natural undamped frequency is 2 kHz. Determine the values of inductor L and the quality factor. 15
- (c) Using Planck's radiation law, deduce Wien's displacement law. How does this law enable one to estimate the surface temperature of the Sun or a star? 15
8. (a) In the circuit given below, find the values of currents I_1 , I_2 and I .
 [Figure: A two-loop d.c. network with three vertical branches between a top wire and a bottom wire. Left branch (top to bottom): $6\ \Omega$ resistor, then a 12 V battery (long plate, i.e. positive terminal, on the upper side). Middle branch (top to bottom): a 3 V battery (long plate, i.e. positive terminal, on the upper side), then plain wire; current I is marked by an upward arrow in this branch. Right branch: $5\ \Omega$ resistor. Top wire: from the top-left corner a $9\ \Omega$ resistor runs to the top of the middle branch; from there a 6 V battery (long plate, i.e. positive terminal, on the left side, towards the middle branch) runs to the top-right corner. Bottom wire: current I_1 is marked by an arrow pointing left (from the bottom of the middle branch towards the left branch) and current I_2 by an arrow pointing right (towards the right branch).] 15
- (b) What are the characteristic features of Rayleigh scattering? A very thin monochromatic beam of light is incident on a particle. Suggest a simple experimental method to ascertain whether the scattering by the particle is of Rayleigh type. 20
- (c) For a Van der Waals gas, write down the equation of state. Determine the coefficient of critical expansion β . 15