

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION 'A'

1. Answer all of the following:

10×5=50

- (a) The density inside a solid sphere of radius a is given by $\rho = \rho_0 a/r$, where ρ_0 is the density at the surface and r denotes the distance from the centre. Find the gravitational field due to this sphere at a distance $2a$ from its centre.
- (b) If I' and I be the Moments of Inertia of a body about an axis passing through an arbitrary origin and about a parallel axis through the centre of mass respectively, show that $I' = MR^2 + I$, where $\mathbf{R}$ is the position vector of the centre of mass with respect to the arbitrary origin and M is the mass of the body.
- (c) A sphere of radius R moves with velocity $\mathbf{u}$ in an incompressible, non-viscous ideal fluid. Calculate the pressure distribution over the surface of the sphere. Do you think that a force is necessary to keep the sphere in uniform motion?
- (d) What is the physical significance of Einstein's A-coefficient? Explain why it is more difficult to achieve Lasing action at X-ray wavelength than at infra-red wavelength.
- (e) For a multimode step index optical fibre, the core refractive index is 1.5 and fractional index difference is 0.001. Calculate the pulse broadening for 1 km length of the fibre. Over a length of 2 km of the fibre, calculate the minimum pulse separation that can be transmitted without overlap.

2. (a) Consider a rigid body rotating about an axis passing through a fixed point in the body with an angular velocity $\boldsymbol{\omega}$. Determine the kinetic energy of such a rotating body in a coordinate system of principal axis. If the Earth suddenly stops rotating, what will happen to the rotational kinetic energy? Comment in detail.

25

(b) A body turns about a fixed point. Show that the angle between its angular velocity vector and its angular momentum vector about a fixed point is always acute.

15

(c) Explain the working principle of a 3-level laser with a specific example. Comment on why the third level is needed.

10

3. (a) A mirror is moving through vacuum with a relativistic speed v in the x -direction. A beam of light with frequency ω_i is normally incident (from $x=\infty$) on the mirror. (i) What is the frequency of the reflected light expressed in terms of ω_i , c and v ? (ii) What is the energy of each reflected photon?

25

- (b) In question 3(a), if the average energy flux of the incident beam is P_i (watts/m²), what is the average energy flux of the reflected beam? 15
- (c) In a certain engine, a piston undergoes vertical SHM with an amplitude of 10 cm. A washer rests on the top of the piston. As the motor is slowly speeded up, at what frequency will the washer no longer stay in contact with the piston? 10
4. (a) Discuss the problem of scattering of charged particle by a coulomb field. Hence, obtain an expression for Rutherford scattering cross-section. What is the importance of the above expression? 25
- (b) A charged particle is moving under the influence of a point nucleus. Show that the orbit of the particle is an ellipse. Find out the time period of the motion. 15
- (c) Considering a plane transmission diffraction grating, where d is the distance between two consecutive ruled lines, m as the order number and θ as the angle of diffraction for normal incidence, calculate the angular dispersion $d\theta/d\lambda$ for an incident light of wavelength λ . 10

SECTION 'B'

5. Answer all of the following:

10×5=50

- (a) In a Young double slit experiment, the first bright maximum is displaced by $y = 2$ cm from the central maximum. If the spacing between slits and distance from the screen are 0.1 mm and 1 m respectively, find the wavelength of light.
- (b) Define Enthalpy and show that it remains constant in a throttling process.
- (c) How does holography differ from conventional photography? What are the requirements for the formation and reading of a hologram?
- (d) In deriving radiation laws, we consider a cubical container of volume V containing a photon gas in equilibrium. Calculate the differential number of allowed normal modes of frequency ω .
- (e) Starting from Maxwell's equation, obtain the wave equation for the electric field $\mathbf{E}$ in free space and appropriate wave equation for the electric field $\mathbf{E} = E_z(x, y, z) \hat{z}$.
6. (a) Show that the group velocity is equal to particle velocity. Also prove that the group velocity of the photons is equal to c , the velocity of light. 15
- (b) For initial current conditions $I = I_0$ and $dI/dt = 0$ at $t = 0$, show that the time dependent current in the critical damping case for an LCR circuit is given by
- $$I = I_0(1 + \gamma t/2)e^{-\gamma t/2}$$
- where $\gamma = R/L$, $\omega_0^2 = 1/LC$, $\omega = \sqrt{\omega_0^2 - \gamma^2/4}$ and $\tan\delta = (-\gamma)/2\omega$. 20
- (c) Using Ampere's Law and continuity equation, show that the divergence of the total current density is zero. 15
7. (a) State and explain Stefan-Boltzmann Law. Show that $\log P = \log K + 4\log R$, where P is the power emitted by black body and R is the resistance of the black body, K is a constant. 10
- (b) One kg of water at 20°C is converted into ice at -10°C at constant pressure. Heat capacity of water is 4,200 J/kg.K and that of ice is 2,100 J/kg.K. Heat of fusion of ice at 0°C is 335×10^3 J/kg. Calculate the total change in entropy of the system. 15
- (c) When connected in series, L_1, C_1 have the same resonant frequency as L_2, C_2 also connected in series. Prove that if all these circuit elements are connected in series, the new circuit will have the same resonant frequency as either of the circuits first mentioned. 15
- (d) Show that the energy flow due to a plane electromagnetic wave propagating along z -direction in a dielectric medium is given by 10

$$\hat{z} k/\omega \mu E_0^2 \cos^2(kz - \omega t),$$

where k and ω are the propagation vector and angular frequency, E_0 is electric field amplitude, μ is the relative permeability of the medium.

8. (a) Consider a system of free gas particles having f degrees of freedom. Use equipartition theorem to establish the relation

$$f = 2((C_p/C_V) - 1),$$

where C_p and C_V are molar specific heats at constant pressure and constant volume respectively. Obtain the values of C_p/C_V for diatomic and triatomic gases. 15

- (b) Show that both Fermi-Dirac and Bose-Einstein distribution functions at an energy E are given by:

$$f(E) \approx \exp[(\mu - E)/k_B T],$$

where $f(E)$ is much smaller than unity, μ and $k_B T$ are the chemical potential and thermal energy of the atom. 10

- (c) Explain the four thermodynamic relations of Maxwell. Using the same, obtain the Clausius-Clapeyron equation

$$dP/dT = L/(T(V_2 - V_1)). \quad \text{15}$$

- (d) Using Maxwell-Boltzmann distribution law prove that there cannot be any negative absolute temperature. 10