

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 250

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

SECTION 'A'

1. Answer all the five parts given below:

10×5=50

- (a) Show that the kinetic energy and angular momentum of the torque free motion of a rigid body is constant.

- (b) Suppose that an S'-frame is rotating with respect to a fixed frame having the same origin. Assume that the angular velocity ω of the S'-frame is given by

$$\omega = 2t \hat{i} - t^2 \hat{j} + (2t+4) \hat{k}$$

where t is time and the position vector $\mathbf{r}$ of a typical particle at time t as assumed in S'-frame is given by

$$\mathbf{r} = (t^2+1) \hat{i} - 6t \hat{j} + 4t^3 \hat{k}.$$

Calculate the Coriolis acceleration at $t = 1$ second.

- (c) Show that a particle of rest mass m_0 , total energy E and linear momentum $\mathbf{p}$ satisfies the relation

$$E^2 = c^2 p^2 + m_0^2 c^4$$

where c is the velocity of light in free space.

- (d) During an earthquake, a horizontal shelf moves vertically. If its motion can be regarded simple harmonic, calculate the maximum value of amplitude of oscillation so that the books resting on it stay in contact with it always. Take $g = 9.8 \text{ ms}^{-2}$ and $T = 0.5 \text{ s}$.

- (e) Explain why information carrying capacity of an optical fibre can be enhanced by reducing the pulse dispersion. How does one minimize pulse dispersion using a graded index optical fibre?

2. (a) If the forces acting on a particle are conservative, show that the total energy of the particle which is the sum of the kinetic and potential energies is conserved. **20**

- (b) Prove that as a result of an elastic collision of two particles under non-relativistic regime with equal masses, the scattering angle will be 90° . Illustrate your answer with a vector diagram. **5**

- (c) Calculate the horizontal component of the Coriolis force acting on a body of mass 0.1 kg moving northward with a horizontal velocity of 100 ms^{-1} at 30° N latitude on the Earth. **15**

- (d) Derive the relativistic length contraction using Lorentz transformation. **10**

3. (a) The dispersion relation for deep water waves is given by

$$\omega^2 = gk + ak^3$$
where g and a are constants. Obtain expressions for phase velocity and group velocity in terms of the wavelength λ . ω and k represent the angular frequency and wave number, respectively. 15
- (b) A parallel beam of light from a He – Ne laser ($\lambda = 630 \text{ nm}$) is made to fall on a narrow slit of width $0.2 \times 10^{-3} \text{ m}$. The Fraunhofer diffraction pattern is observed on a screen placed in the focal plane of a convex lens of focal length 0.3 m . Calculate the distance between the (i) first two minima and (ii) first two maxima on the screen. 15
- (c) The displacement associated with a three-dimensional plane wave is given by

$$\Psi(x, y, z, t) = a \cos[\sqrt{3}/2 kx + 1/2 ky - \omega t].$$
Calculate the angles made by the propagating wave with the x , y and z -axes. 10
- (d) Explain the physical significance of resolving power of a grating with relevant mathematical expression. 10
4. (a) A particle describes a circular orbit under the influence of an attractive central force directed towards a point on the circle. Show that the force varies as the inverse fifth power of distance. 15
- (b) A particle of rest mass $M = 4 \times 10^{-27} \text{ kg}$, disintegrates into two particles of rest masses $M_1 = 3 \times 10^{-27} \text{ kg}$ and $M_2 = 1 \times 10^{-27} \text{ kg}$. Show that the energies E_1 and E_2 of these two parts after disintegration satisfy the condition $E_1 = 3E_2$ while moving in opposite directions with equal linear momenta. Give necessary mathematical derivation. 15
- (c) Show that the operator $(\nabla^2 - 1/c^2 \partial^2/\partial t^2)$ is invariant under Lorentz transformations. 20

SECTION 'B'

5. Answer all the five parts given below: 10×5=50
- (a) In the circuit diagram shown below, calculate the current passing through the milliammeter.
[Figure: Circuit with four corner nodes: S (top left), R (top right), Q (bottom right), P (bottom left). Branch P–S (left side): a 5 V cell, longer (positive) plate on the P side, shorter plate on the S side; arrow pointing upward from P towards S. Branch S–R (top): 200 Ω resistor; current $I - I_1$ marked with arrow from S towards R. Branch R–Q (right side): 60 Ω resistor; current I marked with arrow from R down towards Q. Branch Q–P (bottom): a 4 V cell, longer (positive) plate on the P side, shorter plate on the Q side; current I marked with arrow from Q towards P. Diagonal branch P–R: milliammeter (mA) of resistance 20 Ω ; current I_1 marked with arrow from P towards R.]
- (b) Consider the equation for a series RLC circuit and compare this to the parallel resonant circuit shown below:
[Figure: Parallel resonant circuit of three vertical branches connected between a common top wire and a common bottom wire: left branch a capacitor C , middle branch a resistor R_p , right branch an inductor L . Mesh current I_1 is marked above the left mesh (C – R_p) and mesh current I_2 above the right mesh (R_p – L); a curved arrow in each mesh indicates the circulating current (left mesh clockwise, right mesh anticlockwise).]
Calculate the value of R_p if a series RLC circuit and the parallel RLC circuit are to have same equations for the potential of capacitance while they both have the same L , C and Q with Q being the total charge.
- (c) A thermally insulated ideal gas is compressed quasi-statically from an initial state with volume V_0 and pressure P_0 to a final state of volume V_f and pressure P_f . Show that the work done on the gas in the process is given by

$$W = C_v / R (P_f V_f - P_0 V_0)$$
where C_v and R having standard meanings.

- (d) In a tungsten filament lamp, thermionic emission takes place at 1.2×10^3 K. Calculate the ratio of spontaneous emission to stimulated emission for non-degenerate energy levels. Interpret your result physically. Take $\lambda = 550$ nm, $k_B = 1.38 \times 10^{-23}$ JK⁻¹, $h = 6.67 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms⁻¹.

- (e) The electric field of a plane e.m. wave travelling along the z-axis is

$$\mathbf{E} = (E_{ox} \hat{x} + E_{oy} \hat{y}) \sin(\omega t - kz + \phi).$$

Determine the magnetic field.

6. (a) A series LCR circuit has resonant frequency ω_0 and a large quality factor Q. Write down in terms of R, ω , ω_0 and Q, its (i) impedance at resonance, (ii) impedance at half-power points and (iii) the approximate forms of its impedance at low and high frequencies. 15

- (b) Consider the following coupled inductor – capacitor circuit:

[Figure: Two-mesh ladder circuit. Top wire: inductor L in the left mesh and inductor L in the right mesh, in series along the top. Three vertical branches between the top and bottom wires: left branch capacitor C, middle (shared) branch capacitor C_1 , right branch capacitor C. Bottom wire plain. Thus each mesh contains L, C and the common coupling capacitor C_1 .]

Calculate the ratio of the frequencies of the anti-symmetric and symmetric modes ω_a/ω_s . (Given $k=1/LC$, $k'=1/LC_1$) 10

- (c) Using the fundamental concepts of electromagnetism, determine the electric field of an electric dipole $\mathbf{p}$ at a distance $\mathbf{r}$ and its energy in an electric field $\mathbf{E}$. 15

- (d) ABCD is a rectangle in which charges of $+10^{-11}$ C, -2×10^{-11} C and 10^{-11} C are placed at corners B, C and D, respectively.

[Figure: Rectangle ABCD with A at bottom left, B at bottom right, C at top right, D at top left; AB (horizontal) = 4 cm, AD (vertical) = 3 cm.]

Calculate the potential at the corner A and the work done in carrying a charge of 2 coulombs to A. 10

7. (a)(i) Considering an isotropic, linear, non-conducting, non-magnetic and inhomogeneous dielectric medium with

$$\mathbf{D} = \epsilon \mathbf{E} = \epsilon_0 n^2(x, y, z) \mathbf{E},$$

show that the electromagnetic wave equation for the field $\mathbf{E}$ is given by

$$\nabla^2 \mathbf{E} + \nabla(1/n^2 \nabla \cdot \mathbf{E}) - \mu_0 \epsilon_0 n^2 \partial^2 \mathbf{E} / \partial t^2 = 0. \quad 15$$

- (a)(ii) Write down the scalar equation for E_x from the above equation. 5

- (a)(iii) Interpret physically the situation if we move from homogeneous to an inhomogeneous medium. 5

- (a)(iv) Obtain the similar vector equation for the magnetic field $\mathbf{H}$ in inhomogeneous medium. 15

- (b) For a uniform wire of length L and radius a having a potential difference V between the ends and a current I along it, calculate the energy per unit time delivered to the wire by Poynting vector. 10

8. (a) The vapour pressure, in mm of Hg, of a substance in solid state is given by the relation $\ln p = 23.03 - 3754/T$, where T is in Kelvin. The vapour pressure, in mm of Hg, of the substance in liquid state is given by the relation $\ln p = 19.49 - 3063/T$. Calculate (i) the coordinates of the triple point, and (ii) the latent heat of vaporisation at the triple point. Take Gas constant $R = 8.314$ J mol⁻¹ K⁻¹. 15

- (b) In Leh, temperature of ice on a cold winter night is measured as -20°C . Calculate the change in entropy when 1 kg of ice is converted into steam at 100°C . Given specific heat capacity of ice is 500 cal kg⁻¹ K⁻¹, latent heat of ice is 3.36×10^5 J kg⁻¹, latent heat of steam is 2.26×10^6 J kg⁻¹ and $J = 4.2$ J cal⁻¹. 15

- (c) The coefficient of viscosity of helium at 27°C is 2×10^{-5} kg m⁻¹ s⁻¹. Calculate (i) the average speed and (ii) the diameter of a helium molecule, if it is assumed that the gas obeys Maxwell – Boltzmann distribution. Given Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J 10

K^{-1} and mass of helium atom $= 6.67 \times 10^{-27} \text{ kg}$.

- (d) N particles obeying Classical Statistics are distributed among three states having energies $\epsilon_1=0$, $\epsilon_2=k_B T$ and $\epsilon_3=2k_B T$, where k_B is Boltzmann constant. If the total equilibrium energy of the system is $1000 k_B T$, calculate the value of N.

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