

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer all the five parts below:

12×5=60

(a) Two identical relativistic particles of rest mass m and kinetic energy T collide head-on. What is the relative kinetic energy, i.e., the kinetic energy T' of one in the rest frame of the other?

(b) Define a conservative field. Determine if the field given below is conservative in nature:

$$\mathbf{E} = c[y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}] \text{ V/m}$$

where c is a constant.

(c) Calculate the moment of inertia of a solid cone of mass M , height h , vertical half-angle α and radius of its base R , about an axis passing through its vertex and parallel to its base.

(d) A resistance R and a lossless capacitor C are connected through a switch. The capacitor is charged to potential V_0 , and the switch is closed at $t=0$. Prove that the energy stored in the capacitor is equal to the energy dissipated in the resistor.

(e) A rigid body is spinning with an angular velocity of 4 rad/s about an axis parallel to the direction $(4\hat{i} - 3\hat{k})$ passing through the point A with $\mathbf{OA} = 2\hat{i} + 3\hat{j} - \hat{k}$, where O is the origin of the coordinate system. Find the magnitude and direction of the linear velocity of the body at point P with $\mathbf{OP} = 4\hat{i} - 2\hat{j} + \hat{k}$.

2. The volume between two concentric conducting spherical surfaces of radii a and b ($a < b$) is filled with an inhomogeneous dielectric with $\epsilon = \epsilon_0/(1+cr)$, where c is a constant and r is the radial coordinate. A charge $+Q$ is placed on the inner surface, while the outer surface is grounded. Determine—

(a) $\mathbf{D}$ in the region $a < r < b$; 15

(b) capacitance of the device; 15

(c) polarization charge density in the region $a < r < b$; 15

(d) surface polarization charge densities at $r=a$ and $r=b$. 15

3. A particle is moving in a central force field on an orbit given by $r = ke^{\alpha\theta}$, where k and α are positive constants, r is the radial distance and θ is the polar angle.

(a) Find the force law for the central force field. 20

(b) Find $\theta(t)$. 20

(c) Find the total energy. 20

4. (a) What is a displacement current? Prove that lines of conduction current plus displacement current are continuous. 15
- (b) Consider a uniform half-sphere of radius R and mass M . The half-sphere is supported by a frictionless horizontal plane as shown in the figure. The half-sphere lies in the region $z < 0$.
 [Figure: Solid hemisphere with its flat circular face horizontal and uppermost (the rim drawn as an ellipse, back half dashed) and its curved surface below. Origin O at the centre of the flat face; the z -axis points vertically upward from O and the y -axis horizontally to the right through O along the flat face; R marks the radius of the flat face. The lowest point of the curved surface rests on a horizontal supporting plane A (drawn as a thick black slab), below the origin on the vertical axis. Legend: “ O = Origin”, “ A = Supporting plane”].
 Find the centre of mass of the half-sphere. 15
- (c) Assume $E=0$ inside a perfect conductor. Elaborate on any other four electrostatic properties that arise from this property. 15
- (d) A projectile of mass M explodes, while in flight, into three fragments. One fragment of mass $m_1=M/2$ travels in the original direction of the projectile. Another fragment of mass $m_2=M/6$ travels in the opposite direction and the third fragment of mass $m_3=M/3$ comes to rest. The energy E , released in the explosion, is 5 times the kinetic energy of the projectile at explosion. What are the velocities of the fragments? 15

SECTION 'B'

5. Answer all the five parts below:

12×5=60

- (a) The motion of a damped mechanical oscillator is represented by

$$m\ddot{x} + \alpha\dot{x} + \beta x = 0$$
 where m , α and β are constants. The oscillator is critically damped. The system is given an impulse at $x=0$ and $t=0$, resulting in an initial velocity v . After how much time the system experiences maximum displacement?
- (b) Show that a travelling wave on the string, clamped on both the ends, undergoes a phase change of π . Hence obtain the time-independent form of the wave equation representing a standing wave on the string.
- (c) The separation between the slits is 0.5 mm in Young's double-slit experiment. The interference pattern observed on a screen placed 5 m away reveals the location of the first maximum which is 6 mm from the centre of the pattern. Calculate the wavelength of light and separation between second and third bright fringes.
- (d) Show that the Helmholtz free energy of a system never increases in any isothermal-isochoric transformation.
- (e) Consider non-equilibrium situation for a system in which the population inversion has been achieved. Explain that such a system can be treated as if it has negative absolute temperature.
6. (a) Distinguish between Fresnel and Fraunhofer classes of diffraction. Show that the area of each Fresnel half-period zone is same. 20
- (b) A diffraction grating of width 5 cm with slits of width 10^{-4} cm separated by a distance of 2×10^{-4} cm is illuminated by light of wavelength 550 nm. What will be the width of the principal maximum in the diffraction pattern? Would there be any missing orders? 20
- (c) Establish the relation $(\partial T / \partial V)_P = -(\partial P / \partial S)_T$ and then derive $(\partial C_P / \partial P)_T = -T(\partial^2 V / \partial T^2)_P$. Hence show that the heat capacity C_P of an ideal gas is independent of pressure P . 20
7. (a) Derive an expression for intermodal dispersion for a multimodal step-index fibre. 20
- (b) A pulse of $\lambda_0=600$ nm and $\Delta\lambda=10$ nm propagates through a fibre which has a material dispersion coefficient of 50 ps per km per nm at 600 nm. Calculate the pulse broadening in traversing a 10 km length of the fibre. If the pulse width at the input of the fibre is 12 ns, 10

what will be the pulse width at the output of the fibre?

- (c) Show that both Fermi-Dirac and Bose-Einstein distributions reduce under certain condition in a form which gives the total number of particles as

$$N = A \int_0^{\infty} \sqrt{\epsilon} e^{-\beta \epsilon} d\epsilon$$

where A is a constant and $\beta = 1/k_B T$. Further show that this expression is just the same as that obtained from the Maxwellian speed distribution.

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8. (a) Use matrix method to obtain an expression for the focal length of a coaxial combination of two thin lenses having focal lengths f_1 and f_2 separated by distance d.

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- (b) Calculate the minimum thickness of a quartz plate which would behave as a quarter-wave plate for wavelength of light, $\lambda = 6000 \text{ \AA}$. The refractive indices for ordinary and extraordinary rays are $\mu_o = 1.544$ and $\mu_e = 1.553$.

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- (c) The Einstein theory of specific heat of solids gives the expression

$$C_V = 3Nk_B x^2 e^x / (e^x - 1)^2$$

where $x = \theta_E / T$ with θ_E as the Einstein temperature. (i) Mention Einstein's assumptions in deriving it. Also obtain low- and high-temperature limiting expressions for it. (ii) Give schematic plot of $C_V / 3Nk_B$ versus T / θ_E and comment on the validity of expressions in (i) in comparison with experiments.

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