

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer all the six parts below:

10×6=60

(a) With an appropriate diagram, show that in the Rutherford scattering, the orbit of the particle is a hyperbola. Obtain an expression for impact parameter.

(b) Imagine that a rigid body is rotating about a fixed point with angular velocity ω . Assuming that the coordinate axes coincide with the principal axes, if T stands for kinetic energy and G for external torque acting on the body, show that

$$dT/dt = \mathbf{G} \cdot \boldsymbol{\omega}$$

(c) A spaceship measures 50 m in length on ground and it measured a length of 49.7 m in space as observed from the ground. Find out the speed of the spaceship.

(d) Write down the one-dimensional harmonic oscillator differential equation under damping and its solution for the lightly damped condition, with the meanings of symbols. Determine the dependent energy in the lightly damped condition.

(e) The Fraunhofer single-slit diffraction intensity is given by

$$I = I_0 \sin^2 x / x^2$$

where $x = \pi dy / \lambda l$ with l as the distance from slit to source, d the slit width, y the detector distance and λ the wavelength. What is the value of cumulative intensity $\int_{-\infty}^{\infty} I(y) dy$?

(f) A plane wave has the following expression for its electric field:

$$\mathbf{E} = x\hat{E}_{0x} \cos(\omega t - kz + \alpha) + y\hat{E}_{0y} \cos(\omega t - kz + \beta)$$

If the phase difference is defined as $\delta = \beta - \alpha$, under what conditions do we achieve elliptic polarization? What are the conditions for circular polarization?

2. (a) Determine the number of degrees of freedom for a rigid body– (i) moving freely in space of three dimensions; (ii) having one point fixed; (iii) having two points fixed. **30**

(b) Prove that the time taken by the earth to travel over half of its orbit separated by the minor axis remote from the sun is two days more than half a year. Given, the period of the earth is 365 days and eccentricity of the orbit = 1/60. **10**

(c) With an appropriate diagram, deduce the velocity profile for streamline flow of a liquid through a capillary of circular cross-section. Deduce also the fraction of liquid which flows through the section up to a distance $a/2$ from the axis, where a is the radius of the capillary. **20**

3. (a) Explain the physical significance of group velocity from the concept of phase velocity with relevant expressions. **15**

- (b) Prove that the group velocity V_g of electromagnetic waves in a dispersive medium with refractive index $n(\lambda_0)$ at wavelength λ_0 is given by

$$V_g = c / (n(\lambda_0) - \lambda_0 \frac{dn(\lambda_0)}{d\lambda_0})$$
 where c is the free space velocity of light. Find the time taken for the electromagnetic pulse to travel a distance D . 20
- (c) When a thin film of a transparent material is put behind one of the slits in Young's double-slit interference experiment, the zero-order fringe moves to the position previously occupied by the fourth-order bright fringe. The index of refraction of the film is $n=1.2$ and the wavelength of light, $\lambda=5000 \text{ \AA}$. Determine the thickness of the film. 10
- (d) In relation to a plane diffraction grating having 5000 lines per cm and irradiated by light of wavelength $6000 \text{ \AA}$, answer the following: (i) What is the highest order spectrum which may be observed? (ii) If the width of opaque space is exactly twice that of the transparent space, which order of spectra will be absent? 15
4. (a) For calcite, the refractive indices of ordinary and extraordinary rays are 1.65836 and 1.48641 at $\lambda_0=5893 \text{ \AA}$ respectively. A left circularly polarized beam of this wavelength is incident normally on such crystal of thickness 0.005141 mm having its optic axis cut parallel to the surface. What will be the state of polarization of the emergent beam? 15
- (b) Bring out the essential differences between the physical principles of spontaneous and stimulated emission of radiation. Why is it difficult to get efficient lasing action in case of an ideal two-level material system? Can you propose a scheme to enhance efficiency? Discuss. 15
- (c) Show with proper mathematical analysis that the ratio of Einstein's A and B coefficients depends upon the energy separation between the two energy levels participating in the optical transitions. What is the physical significance of A coefficient? Justify the statement, "It is very difficult to develop an X-ray laser". 30

SECTION 'B'

5. Answer all the six parts below: 10×6=60
- (a) Find whether the discharge of a condenser through the inductive circuit is oscillatory when $C=0.1 \text{ \mu F}$, $L=10 \text{ mH}$ and $R=200 \text{ \Omega}$. If it is oscillatory, calculate its frequency.
- (b) The four arms of a Wheatstone bridge have the following resistances:
 $AB=100 \text{ \Omega}$, $BC=10 \text{ \Omega}$, $CD=5 \text{ \Omega}$, $DA=60 \text{ \Omega}$
 A galvanometer of $15 \text{ \Omega}$ resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10 volts is maintained across AC.
- (c) Write down the expressions for Bose-Einstein and Fermi-Dirac distribution functions, and show how Fermi-Dirac distribution leads to the explanation of Pauli's exclusion principle.
- (d) Find out the total electric potential energy of a single spherical object of uniform charge density ρ , total charge Q and radius R .
- (e) A long solenoid has 220 turns/cm; its diameter is 3.2 cm. Inside the solenoid at its centre, we place a 130-turn close-packed coil of diameter 2.1 cm along its axis. The current in the solenoid is increased from zero to 1.5 amperes at a steady rate over a period of 0.16 second. What is the magnitude of the induced e.m.f. that appears in the central coil when the current in the solenoid is being changed?
- (f) Consider Maxwell's equation in differential form in media. For $j=\rho=0$, assume $\epsilon=\epsilon_0 e^{\alpha t}$ and $\mu=\mu_0 e^{\alpha t}$ and show that the relevant wave equation for a plane wave propagating along x-direction is

$$\partial^2 E / \partial x^2 = \mu \partial^2 D / \partial t^2 + \mu \alpha \partial D / \partial t$$

where $\mathbf{E}=E\hat{y}$ and $\mathbf{H}=H\hat{z}$.

6. (a) An electrical circuit consists of a resistance R , inductance L and capacitance C in series. If a charge is put on the capacitor at some instant, determine the condition that V_C , the voltage across the capacitor, is subsequently oscillatory. Derive an expression for the quality factor Q of the circuit by considering the decay of the oscillation, using the result that the amplitude falls by a factor of e in (Q/π) period. 20
- (b) A resistor $R (=6.2 \text{ M}\Omega)$ and a capacitor $C (=2.4 \text{ }\mu\text{F})$ are connected in series and a 12 V battery of negligible internal resistance is connected across their combination. (i) What is the capacitive time constant of this circuit? (ii) At what time, after the battery is connected, does the potential difference across the capacitor become 5.6 V? 10
- (c) Determine the torque experienced by an electric dipole of moment $\mathbf{p}$ if placed in an electric field $\mathbf{E}$ in nonaligned state. Also show that the interaction energy of two dipoles of moments $\mathbf{p}_1$ and $\mathbf{p}_2$ separated by a displacement $\mathbf{r}$ is
- $$U=1/4\pi\epsilon_0 1/r^3[\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \hat{r})(\mathbf{p}_2 \cdot \hat{r})]$$
- assuming the expression for field due to a dipole. 20
- (d) The electric potential of a grounded conducting sphere of radius a in a uniform electric field is given as
- $$\phi(r,\theta)=-E_0 r[1-(a/r)^3 \cos\theta]$$
- Find the surface charge distribution. 10
7. (a) Consider a plane wave travelling along the positive y -direction incident upon a glass of refractive index $n=1.6$. Find the transmission coefficient. 25
- (b) Justify which of the four Maxwell's equations imply that there are no magnetic monopoles. How these equations would have been written if they were? 15
- (c) In deriving the Rayleigh-Jeans law, we count the number of modes dn corresponding to a wave number k for a photon gas in a cubical box. Consider a cubical container of volume V containing such gas in equilibrium. Calculate the differential number of allowable normal modes of frequency ω . 20
8. (a) What led van der Waals to modify the ideal gas equation? Using the concepts of critical temperature T_c , pressure P_c and volume V_c , show that the critical constant for a real gas is $8/3$. 20
- (b) Find out the expressions for van der Waals constants a and b . 20
- (c) Calculate the values of van der Waals constants a and b for oxygen with $T_c=154.2 \text{ K}$, $P_c=49.7 \text{ atmosphere}$ and $R=80 \text{ cm}^3 \text{ atmosphere/K}$. 20