

PHYSICS (Paper I)

Time Allowed : Three Hours

Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer all the six below:

10×6=60

- (a) A uniform solid sphere of radius R having moment of inertia I about its diameter is melted to form a uniform disc of thickness t and radius r . The moment of inertia of the disc about an axis passing through its edge and perpendicular to the plane is also equal to I . Show that the radius r of the disc is given by $r=2R/\sqrt{15}$.
- (b) Two thin symmetrical lenses of two different natures (convex and concave) and of different materials have equal radii of curvature $R=15$ cm. The lenses are put close together and immersed in water ($\mu_w=4/3$). The focal length of the system in water is 30 cm. Show that the difference between the refractive indices of two lenses is $1/3$.
- (c) What is the significance of the null result of Michelson-Morley experiment? Does it disprove the existence of ether? Justify.
- (d) An optical beam of spectral width 7.5 GHz at wavelength $\lambda=600$ nm is incident normally on Fabry-Perot etalon of thickness 100 mm. Taking refractive index unity, find the number of axial modes which can be supported by the etalon.
- (e) An unpolarized light beam of intensity 1000 W/m^2 is incident on an ideal linear polarizer with its transmission axis parallel to vertical direction. Describe an experiment to reduce the intensity of light beam to 500 W/m^2 .
- (f) What should be the refractive index of cladding of an optical fibre with numerical aperture 0.5 with refractive index of core as 1.5?

2. (a) What are Eulerian angles? A body with rotational symmetry about an axis is rotating under gravity about a point on the axis without friction. What are the quantities remaining constant during the motion? Find them in terms of suitable Eulerian angles. Explain 'precession' and 'nutation' of such a body.

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(b)(i) A planet revolves around the Sun in an elliptic orbit of eccentricity e . If T is the time period of the planet, find the time spent by the planet between the ends of the minor axis close to the Sun.

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(b)(ii) When a sphere of radius r falls down a homogeneous viscous fluid of unlimited extent with the terminal velocity v , the retarding viscous force acting on the sphere depends on the coefficient of viscosity η , the radius r and its velocity v . Show how Stokes' law was arrived at connecting these quantities from the dimensional considerations.

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- (c) In the propagation of longitudinal waves in a fluid contained in an infinitely long tube of cross-section A, show that

$$\rho = \rho_0(1 - \partial\xi/\partial x)$$

where, ρ_0 = equilibrium density, ρ = density of the fluid in the disturbed state, $\partial\xi/\partial x$ = volume strain ($|\partial\xi/\partial x| \ll 1$)

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3. (a) If a photon of frequency ν collides with an electron of mass m_0 at rest, it will be scattered at some angle θ with a new frequency ν' . Show that the change in wavelength is related to the scattering angle by the formula

$$\lambda' - \lambda = 2\lambda_C \sin^2(\theta/2)$$

where $\lambda_C = h/m_0c$ is known as the Compton wavelength.

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- (b) Show that two convex lenses of the same material kept separated by a distance a , which is equal to the average of two focal lengths, may be used as an achromat, that is, $a = 1/2(f_1 + f_2)$.

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- (c) Describe Michelson interferometer for evaluation of coherence length of an optical beam. Calculate coherence length of a light beam of wavelength 600 nm with spectral width of 0.01 nm.

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4. (a) Obtain the expression for the primary focal length of Fresnel zone plate.

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- (b) Show that two light beams polarized in perpendicular directions will not interfere.

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- (c) A laser beam of 1 micrometer wavelength with 3 megawatts power of beam diameter 10 mm is focussed by a lens of focal length 50 mm. Evaluate the electric field associated with the light beam at the focal point. (Dielectric permittivity of free space, $\epsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/\text{N-m}^2$)

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SECTION 'B'

5. Answer all the six below:

10×6=60

- (a) Obtain Poisson's equation in electrostatics from Gauss' law. What form does it take when the charge density is zero?

- (b) A wire of length 2 m is perpendicular to X-Y plane. It is moved with a velocity $\mathbf{V} = (2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \text{ m s}^{-1}$ through a region of uniform induction $\mathbf{B} = (\mathbf{i} + 2\mathbf{j}) \text{ W m}^{-2}$. Compute the potential difference between the ends of the wire.

- (c) What happens if the primary winding of a transformer is connected to a battery?

- (d) 1 kmol of an ideal gas is compressed isothermally at 400 K from 100 kPa to 1000 kPa in a piston and cylinder arrangement. Calculate the entropy change of the gas, the entropy change of the surroundings and the total entropy change resulting from the process if the process is mechanically reversible and the surroundings consist of a heat reservoir at 400 K.

- (e) Calculate the change in pressure for a change in freezing point of water equal to -0.91°C . Given, the increase of specific volume when 1 gm of water freezes into ice is 0.091 cc/gm and latent heat of fusion of ice is 80 cal/gm.

- (f) Calculate the number of different arrangements of 10 indistinguishable particles in 15 cells of equal a priori probability, considering that one cell contains only one particle.

6. (a) What is meant by a dielectric? Define polarization vector $\mathbf{P}$ and relate it with the average molecular dipole moment. Obtain expression for the potential due to a polarized dielectric in terms of the polarization vector.

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- (b) Discuss the growth of current when an e.m.f. is suddenly applied to a circuit containing resistance, inductance and capacitance in series. What is the time constant of the circuit?

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- (c) A series circuit has an inductance of 200 microhenries, a capacitance of 0.0005 microfarad and a resistance of 10 ohms. Find the resonant frequency and quality factor of the circuit. **20**
7. (a) Using Maxwell's field equations for a homogeneous non-conducting medium, derive the wave equation for the electric field. Calculate the velocity of EM wave in free space. **20**
- (b) Explain the term 'Poynting vector' and state the significance of Poynting theorem. **20**
- (c) Calculate the skin depth for radio waves in free space of wavelength 3 m in copper, given that electrical conductivity for copper is $6 \times 10^7 \Omega^{-1} \text{ m}^{-1}$. **20**
8. (a) Consider one mole of an ideal gas whose pressure changes with volume as $P = \alpha V$, where α is a constant. If it is expanded such that its volume increases m times, find the change in internal energy, work done by the gas and heat capacity of the gas. **25**
- (b) Derive an expression for the thermal efficiency of a reversible heat engine operating on the Diesel cycle with an ideal gas of constant heat capacity as the working medium. **25**
- (c) Consider the following statement: "The Fermi energy of a given material is the energy of that quantum state which has the probability equal to 1/2 of being occupied by the conduction electrons." Is the above statement correct? Give reasons for your answer. **10**