

PHYSICS

(Paper II)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. (a) Using dimensional analysis, explain why the angular momentum of a particle cannot be $\hbar^2$. 10

- (b) (i) Establish that

$$hc = 1240 \text{ eV} \cdot \text{nm} = 1240 \text{ MeV} \cdot \text{fm}$$

- (ii) The energy levels of a hydrogen atom are given by

$$E_n = (-1/n^2) \text{ Ryd},$$

where

$$1 \text{ Ryd} = hcR.$$

Show that

$$R = 1.097 \times 10^7 \text{ m}^{-1}.$$

What exactly is R? 10

- (c) Identify all the spin-half particles in the following list: Antineutron, Antiproton, Muon, Muon Neutrino, Neutral Kaon, Neutral Sigma, Neutral Xi, Neutron, Photon, Positive Pion, Positron, Proton, u-quark. 10

- (d) (i) Find the ground state total orbital (L) and total spin (S) quantum numbers for nitrogen. 10
(ii) Find the L and S values of the two "first excited states" of helium atom. 10

- (e) Explain fluorescence and phosphorescence in electronically excited molecules. 10

- (f) $2^2S_{1/2}$ level in H atom is 1058 MHz above the $2^2P_{1/2}$ level. (i) What is this known as? (ii) Express the above frequency in cm^{-1} . (iii) Calculate the energy difference between the above two levels in eV. 10

2. (a) Show that the Pauli Spin Matrices obey the following relations: (i) $\text{Tr}(\sigma_x) = \text{Tr}(\sigma_y)$ (ii) $\det(\sigma_y) = \det(\sigma_z)$ (iii) The eigenvalues of σ_z and σ_x are the same. (iv) Write down the y-component of the spin angular momentum matrix corresponding to an antineutrino. 20

- (b) (i) The quantum mechanical probability distribution function of an electron in the ground state of the hydrogen atom is

$$P(r) = Nr^2 \exp(-2br).$$

Using the result

$$\int_0^\infty P(r) dr = 1,$$

deduce that N is proportional to b^3 . (ii) Prove that the value of $40 k_B T$ at $T = 300 \text{ K}$ is nearly 1 eV. Hence determine the Fermi temperature of a metal whose Fermi energy is 9.4 eV. (iii) Show that the Fermi velocity is related to the Fermi energy of electrons through the relation $v_F/c = 1.98(E_F/1 \text{ MeV})^{1/2}$. 20

- (c) Show that the time-dependent part of all the solutions of the Schrödinger equation in one-dimension has the structure

$$\phi(t) = \exp(-iEt/\hbar),$$

provided the potential is not an explicit function of time.

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3. (a) (i) Consider a positron in a box. If the energy released is 60 eV when it jumps from the third excited state to the ground state, show that the width of the potential is nearly 0.3 nm. (ii) Prove that the most probable distance of an electron from the proton (in the hydrogen atom) is the Bohr radius of the hydrogen atom. Consider only the ground state.

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- (b) (i) Consider a particle in a three-dimensional box. Derive an expression for $g(E)$, the density of states. (ii) Show that

$$g(p)/g(E) = dE/dp,$$

where $g(p)$ is the density of states in the momentum space. Deduce that $g(p)$ is proportional to p^2 for a free non-relativistic particle.

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- (c) What was the aim of Stern – Gerlach experiment? Why were silver atoms chosen instead of electrons in the experiment? What was the outcome of the experiment?

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4. (a) Discuss the vibrational spectra of a diatomic molecule treating it as a harmonic oscillator as well as an anharmonic oscillator and compare them.

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- (b) (i) Explain spin-orbit coupling of an atomic electron. (ii) Show that the 2p state in the H atom splits up into two substates due to spin-orbit coupling. (iii) Calculate the energy of separation in eV, resulting from the spin-orbit coupling when the magnetic field experienced by the electron is 0.4 T.

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- (c) Explain Born – Oppenheimer approximation. Discuss the intensity distribution in the vibrational electronic spectra of a diatomic molecule on the basis of Franck – Condon principle.

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SECTION 'B'

5. (a) What are the field quanta of weak and electromagnetic interactions? Show that weak interaction is a short range force while electromagnetic interaction is a long range one.

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- (b) Write down the Yukawa potential and derive an expression for the mass of π -meson in terms of range of nuclear force. What part of the Nucleon – Nucleon interaction is explained by this potential?

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- (c) Show that the reciprocal lattice of a hexagonal lattice is a hexagonal lattice with a rotation of axes.

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- (d) A type-II superconducting material with superconducting transition temperature T_c is placed in a magnetic field. What is the material state (normal or superconducting or mixed) for (i) $T < T_c$; $H > H_{c_2}$ (ii) $T > T_c$; $H < H_{c_1}$ (iii) $T < T_c$; $H_{c_1} < H < H_{c_2}$ (iv) $T < T_c$; $H < H_{c_1}$ (v) $T > T_c$; $H > H_{c_2}$ where H_{c_1} and H_{c_2} are the lower and upper critical fields for a type-II superconductor.

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- (e) Verify that the circuit shown below is an exclusive OR.

[Figure: Logic circuit of four two-input NAND gates with inputs A and B and output Y. Gate 1 has inputs A and B; its output is labelled Y_1 . Gate 2 (upper) has inputs A and Y_1 ; its output is labelled Y_2 . Gate 3 (lower) has inputs Y_1 and B; its output is labelled Y_3 . Gate 4 has inputs Y_2 and Y_3 ; its output is Y.]

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- (f) Silicon is the most popular semiconductor material used for the fabrication of electronic devices and integrated circuits. Why? Give at least five reasons.

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6. (a) What is the simplest two-nucleon bound system? How does its study help in obtaining information about nuclear forces? Indicate clearly how the non-central forces can be explained by the observed magnetic moment of this system.

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- (b) At what energies, do we anticipate the unification of strong and electroweak interactions? What will happen to coupling constants of these interactions in this situation? How does

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unification indicate the decay of proton and existence of leptoquark?

- (c) Compare the methods of electron scattering and neutron scattering experiments to obtain information about nuclear size. 20
7. (a) Which of the waves out of (i) X-rays, (ii) de-Broglie waves due to electrons or neutrons will be suitable for crystal diffraction? Justify your answer in terms of energies of these waves for 0.1 nm. 15
- (b) Derive an expression for the specific heat of a solid on the basis of Debye's model. Show that it converges to Dulong and Petit's law at high temperatures. 25
- (c) What is Meissner effect? Explain the conclusions drawn from the Meissner effect. 20
8. (a) List the characteristics of an ideal OP-AMP. Write the expression for the output voltage in terms of input voltages and resistance values. 20
- [Figure: Inverting summing amplifier. Three input terminals v_1, v_2, v_3 are connected through resistors R_1, R_2, R_3 respectively to a common junction, which goes to the inverting ($-$) input of the op-amp. A feedback resistor R_0 connects the op-amp output back to the inverting input. The non-inverting (+) input is connected to the common (ground) line running along the bottom of the circuit from a ground input terminal to the lower output terminal. The output voltage v_o is taken between the op-amp output terminal and the common line (arrow between the two output terminals).]
- (b) The resistance of silicon and germanium varies with temperature. But these are not used normally as thermistors. Why? Name at least two widely used materials for making thermistors and give any five applications of thermistor. 20
- (c) What do you understand by early effect in bipolar junction transistors? What are its consequences? Discuss briefly the advantage of using the BJT in the common collector configuration. 20