Civil Services (Main) Examination, 2009

PHYSICS

(Paper I)

Time Allowed : Three Hours **Maximum Marks : 300**

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.
Candidate has to attempt FIVE questions in all.
Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.
The number of marks carried by a question/part is indicated against it.
Wherever any assumptions are made for answering a question, they must be clearly indicated.
Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. (a) Show that the total energy per unit mass of liquid flowing from one point to another without any friction remains constant throughout the displacement. **10**

(b) Let

$$E_A=E_1\sin\omega t \text{ and } E_B=E_2\sin(\omega t+\delta)$$

By using analytical method, obtain an expression to explain interference. Also show that intensity varies along the screen in accordance with the law of cosine square in interference pattern. **10**

(c) Two bodies of masses M_1 and M_2 are placed at a distance d apart. Show that at this position where the gravitational field due to them is zero, the potential is given by

$$V=-G/d(M_1+M_2+2\sqrt{(M_1M_2)})$$ **10**

(d) Show that a four-dimensional volume element $dx\ dy\ dz\ dt$ is invariant to Lorentz transformation. **10**

(e) Derive the condition for achromatism of two thin lenses separated by a finite distance and made up of same material. **10**

(f) What are the characteristics of stimulated emission? Show that in the optical region, stimulated emission is negligible compared to spontaneous emission. **10**

2. (a) Consider a spherical shell of mass M and radius R. Calculate the potential due to this shell at a point P when the point is (i) outside the shell and (ii) inside the shell ($r<R$). If the spherical shell is now replaced by a uniform solid sphere of same mass and radius, what will be its potential at the same external point? **15**

(b) Show that for any rigid body consisting of at least three particles, not arranged in one straight line, number of independent degrees of freedom is six. Define Euler's angles θ, ϕ and ψ to describe the configuration of such a rigid body. Consider two frames of reference, one fixed to the body and the other to the space defined as $S'=(x',y',z')$ and $S=(x,y,z)$ respectively. Show that the angular momentum ($\mathbf{L}$) of the rigid body in the two frames are related by

$$(d\mathbf{L}/dt)_S=(d\mathbf{L}/dt)_{S'}+\boldsymbol{\omega}\times\mathbf{L}$$

where $\boldsymbol{\omega}$ is the angular velocity of rotation. **30**

(c) Obtain Poiseuille's equation for a viscous fluid flowing through a narrow tube of radius r and length l. If a spherical body of radius a is allowed to move at a speed **V** through the same fluid of viscosity η, show that the viscous force will increase with the speed **15**

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linearly.

3. **(a)** For stationary waves on a string whose ends are fixed, show that the energy density is maximum at antinodes and minimum at nodes. **20**

(b) Explain the phenomenon of interference in thin films. Why is the contrast better in brightness of fringes obtained from the interference of reflected light rays compared to the transmitted light rays? **20**

(c) Obtain the relativistic equation for aberration of light using velocity transformation equations. **20**

4. **(a)(i)** Distinguish between high dispersive power and high resolving power. **5**

(a)(ii) Obtain an expression for the resolving power of a plane transmission grating. **10**

(a)(iii) Deduce the missing orders for a double-slit Fraunhofer pattern, if the slit widths are 0.16 mm and 0.8 mm apart. **5**

(b)(i) Show that the plane of polarisation is rotated through

$$\theta=\delta/2=\pi d/\lambda(\mu_L-\mu_R)$$

in optical rotation where symbols have their usual meanings. **15**

(b)(ii) A plane-polarised light is incident perpendicularly on a quartz plate cut with faces parallel to optic axis. Find the thickness of the quartz plate which introduces phase difference of 60° between e- and o-rays. **5**

(c)(i) At what temperature are the rates of spontaneous and stimulated emission equal? (Assume λ=500 nm) **7**

(c)(ii) What are the important properties of a hologram? **6**

(c)(iii) Optical power of 1 mW is launched into an optical fiber of length 100 m. If the power emerging from the other end is 0.3 mW, calculate the fiber attenuation. **7**

SECTION 'B'

5. **(a)** A cylindrical conductor is carrying a current along its axis which is assumed to be in z-direction. The current is uniformly distributed throughout its cross-section. Show that the vector potential **A** associated with the magnetic induction due to the current-carrying cylindrical conductor is independent of z. **10**

(b) State Gibbs phase rule. Show that for a 1-component closed thermodynamic system having two phases, the condition for equilibrium between the phases is that their specific Gibbs functions are equal. **10**

(c) A long solenoid of radius R and n turns per unit length carries a sinusoidal current $I=I_0\cos\omega t$. Determine the magnitude of induced electric field (E) outside the solenoid. **10**

(d) A series R-L-C circuit is connected across a voltage source $V=100\sin 300t$. If R=500 Ω, L=1 H and C=2 μF, calculate the average power delivered to the circuit. **10**

(e) Using Planck's radiation formula, deduce Wien's displacement law. **10**

(f) A thin dielectric cylindrical rod of cross-section A is situated along z-axis from $z=0$ to $z=L$. The polarisation of the rod is along its length and it is given by $\mathbf{P}=(2z^2+5)\hat{z}$. Calculate bound volume charge density at each end of the rod. **10**

6. **(a)** For an arbitrary localised charged distribution, obtain an expression of electrostatic potential V in terms of multipole expansion. **25**

(b) State Biot-Savart law. Calculate the magnitude of axial magnetic induction due to a circular loop of area A carrying current I. **20**

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(c) Consider in the region $0 \le z \le 1$ m an infinite slab made of a material with relative permeability, $\mu_r = 3 \cdot 5$. If

$$\mathbf{B} = (2y\mathbf{i} - 5x\mathbf{j}) \times 10^{-3} \text{ Wb/m}^2$$

within the slab, determine magnetisation **M**. **15**

7. (a) Consider the incidence of a plane-polarised electromagnetic wave at the interface of two media having dielectric permittivity and magnetic permeability (ε_1, μ_1) and (ε_2, μ_2) respectively. The interface is chosen to be $x=0$ plane. $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{K}_3$ represent the propagator vectors associated with the incident, refracted and reflected waves respectively. Using the boundary conditions on them, establish the Snell's laws of refraction. **15**

(b) In a non-charged current-free dielectric, $\rho = 0$ and $\mathbf{J} = 0$. Show that in this medium, electric (**E**) and magnetic (**H**) fields satisfy three-dimensional wave equations

$$\nabla^2 \mathbf{E} = \varepsilon\mu \partial^2 \mathbf{E} / \partial t^2 \text{ and } \nabla^2 \mathbf{H} = \varepsilon\mu \partial^2 \mathbf{H} / \partial t^2$$

Using Poynting theorem of electromagnetic theory, describe the significance of the vector $\mathbf{P} = (\mathbf{E} \times \mathbf{H})$ and the scalar

$$u = 1/2[\mathbf{B} \cdot \mathbf{H} + \mathbf{D} \cdot \mathbf{E}]$$ **20**

(c) A plane-polarised electromagnetic wave is incident on the interface of two dielectrics having dielectric permittivity ε_1 and ε_2. Assume that the electric vector **E** lies in the plane of incidence. Using the boundary conditions at the interface, obtain the expressions for the amplitude reflection coefficient (r_{11}) and the amplitude transmission constant (t_{11}). Using the components of the Poynting vector $\mathbf{E} \times \mathbf{H}$ associated with the reflected and transmitted waves, obtain the expressions for reflection and transmission coefficients R_{11} and T_{11} respectively. Under what condition, $r_{11} = 0$ and $t_{11} = 1$? **25**

8. (a) Define entropy. How is it related to disorder? Hence, derive the Boltzmann relation $S = k \log \Omega$, where Ω is the probability and k is the Boltzmann constant. Show that for any type of process, involving a closed system

$$\Delta S \ge \Delta Q / T$$

where the equality sign applies for internally reversible processes and the inequality for internally irreversible processes. **20**

(b) What are the limitations of Einstein's theory of specific heat of solids when compared with experiments at low temperature? Outline the assumptions made in Debye's theory and show that the specific heat at low temperature follows $C_v \sim T^3$ law. What is the significance of Debye's temperature, T_D? **20**

(c) Energy distribution for n_i particles in classical statistical mechanics is given by

$$n_i = g_i e^{-\alpha - \beta\varepsilon_i}$$

where α and β are constants. g_i is the single particle states in the i th level. Using equipartition theorem, show that correct thermodynamic interpretation is

$$\beta = 1/kT$$

(Use $\int_0^\infty e^{-x} x^{1/2}\, dx = \sqrt{\pi}/2$ and $\int_0^\infty e^{-\beta x} x^{3/2}\, dx = 3\sqrt{\pi}/(4\beta^{5/2})$) **20**

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