

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:

3×20=60

- (a) Write down the Lagrangian of a free particle in rectangular cartesian coordinates. Identify all the cyclic coordinates. Show that the constants of motion obtained from the equations for the considered form of the Lagrangian are not exactly the same as those which follow from the concept of a free particle.
- (b) What are chromatic and spherical aberrations Explain. Calculate the longitudinal aberration for rays at a height of 0.01 m from the principal axis and incident on the curved surface (radius of curvature = 0.1 m) of a thin plano-convex lens made of glass (refractive index = 1.5).
- (c) A body of rest mass m_0 is moving in the positive direction at a velocity of 0.6 C relative to the laboratory frame. Calculate the components of the four dimensional momentum vector in the laboratory frame and in the frame of an observer who is travelling in the positive x-direction at a speed of 0.8 C relative to the laboratory frame.
- (d) Consider superposition of two plane polarized electromagnetic waves:

$$E_y = \hat{y} a \cos(kx - \omega t)$$

and

$$E_z = \hat{z} b \cos(kx - \omega t + \phi)$$

Discuss the conditions for the resultant wave to be left circularly and right circularly polarized adopting the convention as seen by an observer travelling with the wave.

2. (a) Derive the relation $\mathbf{v}_0 = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}$, where $\mathbf{v}_0$ is the velocity of a particle located at $\mathbf{r}$ in a fixed frame of reference S and $\mathbf{v}$ that as observed in a frame S' rotating with angular velocity $\boldsymbol{\omega}$ with respect to S but having the common origin. 25
- (b) Show, using the above relation, that the equation of motion of the particle in S gets modified in S' giving rise to various fictitious forces. Identify the coriolis force and describe its effect on the flow of rivers. 20
- (c) The angular momentum $\mathbf{M}$ of a rigid body comprising of N particles and rotating with angular velocity $\boldsymbol{\omega}$ is given by:

$$\mathbf{M} = \sum_{k=1}^N m_k \mathbf{r}_k \times (\boldsymbol{\omega} \times \mathbf{r}_k)$$

where the origin coincides with the centre of mass. Express the components of $\mathbf{M}$ in terms of components of the inertia tensor. Hence, show that the most general free rotation of a spherical top is a uniform rotation about an axis fixed in space. 15

3. (a) State the postulates of the special theory of relativity and based on these obtain Lorentz as well as inverse Lorentz transformations. Hence, obtain an expression to conclude that a moving clock runs more slowly than a stationary clock. 35
- (b) An unstable particle has a lifetime of $5 \mu\text{s}$ [?] in its own frame of reference and is moving towards the earth at a speed of $0.8 C$. What will be the lifetime of the particle to an observer on the earth? 10
- (c) Establish the relationship between the phase velocity V_p and the group velocity V_g of waves. Under what physical conditions $V_g < V_p$, $V_g = V_p$ and $V_g > V_p$ can be possible? 15
4. (a) Describe the working of a Fabry-Perot interferometer. Determine the intensity of the fringes of the transmitted light. Why the fringes obtained in the Fabry-Perot interferometer are comparatively sharper than those obtained from the Michelson interferometer? 35
- (b) The radius of the first zone in a zone plate is 2 mm. What will be the position of the first image of a point source of light of wavelength $\lambda = 500 \text{ nm}$ placed at a distance of 5 m from the zone plate? 10
- (c) Define resolving power and dispersive power of a grating. Two spectral lines of wavelengths 500 nm and 500.5 nm are seen clearly resolved in second order spectrum of a grating. If the grating has 250 lines per cm, what should be the minimum width of the grating? 15

SECTION 'B'

5. Answer any three of the following: **20×3=60**
- (a) Prove the law of increase of entropy. Show that for a system at fixed temperature and pressure to be in equilibrium, its Gibbs free energy should be minimum.
- (b) A system at temperature T_1 is brought in contact with a reservoir at temperature $T_2 > T_1$. When the system and the reservoir reach thermal equilibrium, calculate the change in entropy of the universe assuming the heat capacity of C_p of the system to be constant. Discuss whether the considered change is positive or not.
- (c) In an ac. circuit, a resistance ($R = 100 \Omega$) and a capacitance ($C = 100 \mu\text{F}$) in series are connected to an a.c. source $V = 200 \sin(100\pi t)$. Calculate the current through the circuit and the voltages across R and C. Draw a vector diagram representing the magnitudes and phases of the voltages.
- (d) State Amperes law of magnetostatics. Using this law, find the magnetic field at a point due to an infinitely long filamentary current.
6. (a) Derive Poisson equation starting from the Coulomb's law for a set of point charges. 20
- (b) Obtain the solution of the Laplace equation in cylindrical coordinates. 20
- (c) What is a magnetic shell? Define the strength of a magnetic shell. A 2 mm thick magnetic shell weighing 100 gm has magnetic moment of 1000 units. The density of the shell material is 10 gm/cc. Calculate the intensity of magnetisation and the strength of the shell. 20
7. (a) Explain the Rayleigh Scattering of light. Show that the energy density of light scattered from an isotropic homogenous medium of a gas is inversely proportional to the fourth power of the wavelength of the incident light. 35
- (b) State Rayleigh-Jeans law. Show that the intensity of emissions at a particular wave-length is proportional to the temperature T. Discuss the limitations of this law in describing the intensity distribution of emission spectrum of a blackbody. 25
8. (a) Establish the relation $C_p - C_v = [P + (\partial U / \partial V)_T](\partial V / \partial T)_P$ 25

Use it to find out an expression for C_p of one mole of a gas whose internal energy is given by $U=cT-a/V$ and which satisfies the equation of state $[P+a/V^2](V-b)=RT$. Here a , b and c are constants.

- (b) Derive a relation between the total number of Fermions in terms of Fermi momentum and hence obtain the expression for the total energy E of the system at absolute zero. Combine this expression with the equation of state $PV=2/3E$ to show that the pressure of an ideal Fermi gas at $T = 0$ is proportional to $5/3$ power of its number density.

35