

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:

3×20=60

- (a) Derive an equation of motion for a variable mass. Explain how it is applied in the motion of a rocket.
- (b) Prove that two successive Lorentz transformations are equivalent to another Lorentz transformation. Hence write down the Einstein's velocity addition relation.
- (c) The X and Y co-ordinates of Cornu's spiral can be expressed quantitatively by two integrals. Derive the expressions of these integrals.
- (d) What is optical activity? Give reasons for the conclusion that optical rotation in liquids has a molecular origin. What do you mean by ordinary and extraordinary rays? What are positive and negative crystals? Give an example of each. Compute the minimum thickness of a quarter-wave plate made from quartz for incident wavelength of 589.3 nanometer. Given $\mu_o = 1.544$ and $\mu_E = 1.553$.

2. (a) What is Hamilton's principle? Obtain Lagrange's equations of motion with its help for a conservative system. 25

(b) What is centre of mass? Show that the total linear momentum of a system of particles about the centre of mass is zero. 10

(c) Derive Euler's equations of motion for a rigid body rotating about a fixed point under the action of a torque. When a rigid body is not subjected to any net torque, write down Euler's equations of motion of the body with one point fixed. 25

3. (a) The source S' moves along the x' -axis at a speed v , and emits light at an angle θ' to the x' -axis of its own frame. In the S-frame the emitting angle with the x-axis is θ . Here x and x' -axes are coincident. Show that the exact relativistic aberration formula

$$\tan\theta = (\sin\theta' \sqrt{1-v^2/c^2}) / (\cos\theta' + v/c)$$

can be derived from the velocity transformation relations. 25

(b) In the steady-state forced vibration a point particle of mass 'm' moves under the influence of an external force $(F \sin pt)$ $\hat{i}$ in addition to the restoring force $-(kx)$ $\hat{i}$ and damping force $-(\beta \dot{x})$ $\hat{i}$. Show that (i) the amplitude is maximum when $p = \sqrt{(\omega^2 - 2b^2)}$, where $k/m = \omega^2$ and (ii) the value of the maximum amplitude is $f/(2b\sqrt{(\omega^2 - 2b^2)})$. What do you mean by the sharpness of resonance? 15

- (c) Derive Snell's law of refraction from Fermat's principle. How is the refractive index related to the velocity of light? Light of wavelength 600 nm (in vacuum) enters a glass slab of refractive index 1.5. What are the values of wavelength, frequency and velocity of light in glass? [Velocity of light in vacuum = $3 \times 10^8 \text{ ms}^{-1}$] 20
4. (a) Explain the working of Michelson interferometer using appropriate optical diagram. also draw paths of the rays. 20
- (b) Obtain the relation to find radii of the rings and the wavelength of light in Newton's circular ring. Calculate the radius of curvature of the convex glass surface where diameter of 5th and 15th bright rings formed by sodium yellow light are measured to be 2.303 mm and 4.134 mm. Given $\mu = 1.5$ and $\lambda_{\text{yellow}} = 5892 \text{ \AA}$. 20
- (c) How would you produce plane polarized light by reflection? What is Brewster's law? Calculate the angular position of the sun above the horizon so that light reflected from a calm lake is completely polarized. The refractive index of water is 1.33. Circularly polarized and unpolarized light are passed in turn through a Nicol prism. The Nicol is rotated about the direction of light as axis. What would you observe in each case? How would you distinguish between them? 20

SECTION 'B'

5. Answer any three of the following:

20×3=60

- (a) What is molecular polarizability? Derive Clausius – Mossotti equation relating the molecular polarizability with the dielectric constant of a dielectric material.
- (b) Find the magnetic field **B** at the point P due to a short straight length of wire carrying current 'i'. Length of the wire is l. Point P is at a distance r away from the centre of the wire. Angle between **l** and **r** is θ .
- (c) What do you mean by a gauge transformation? What is its importance? Show that the Lorentz gauge condition $\nabla \cdot \mathbf{A} + 1/c \partial \phi / \partial t = 0$ is Lorentz invariant. Here **A** and ϕ are the vector and scalar potentials.
- (d) 1 kg of ice at 0° C floats on 10 kg of water at 30° C, the whole system being thermally isolated. What will be the change in entropy of the system when thermal equilibrium is reached? [Specific heat of water = $4.2 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and latent heat of fusion of ice = 336 kJ kg^{-1}]
6. (a) Starting from Maxwell's equation, $\nabla \cdot \mathbf{D} = \rho$, where **D** is the electric displacement density and ρ is the charge density, derive Poisson's equation. Deduce Laplace's equation for charge-free region from Poisson's equation. 20
- (b) A potential in cylindrical coordinates is a function of r and ϕ but not of z. Obtain the separated differential equations for R and Φ , where $V = R(r) \Phi(\phi)$ and solve them. 20
- (c) Write Ampere's circuital law, and obtain a generalised form of this law, for non-stationary case. 20
7. (a) Starting from Maxwell's equations of electromagnetic field in vacuum obtain the classical wave equations for the four field vectors **E**, **D**, **B** and **H**. Show that the field vectors can be propagated as waves in free space with the velocity of propagation equal to $3 \times 10^8 \text{ m/s}$, where for free space we have the vacuum permittivity $\epsilon_0 = 8.854 \times 10^{-12} \text{ farad/m}$ & vacuum permeability $\mu_0 = 1.257 \times 10^{-6} \text{ henry/m}$. 20
- (b) Write down the different components of the electromagnetic field tensors $F_{\mu\nu}$ and further prove that Maxwell's equations of electrodynamics are invariant to Lorentz transformations. 20
- (c) Using Planck's radiation formula
- $$u(\nu) d\nu = 8\pi h / c^3 \nu^3 d\nu / (e^{h\nu/kT} - 1)$$
- 20

[?] where the symbols have their usual meaning, find the wavelength of the region where energy density is the greatest. Also calculate the total energy density over all the frequencies.

8. (a) Using thermodynamic principles show that the Joule-Thomson coefficient μ can be expressed as

$$\mu = \frac{1}{C_p} [T(\partial V / \partial T)_p - V]$$

Calculate the value of μ for an ideal gas and interpret your result physically.

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- (b) State Dulong and Petit's law. How does it agree with experiment? Discuss the limitations of the classical theory and success of the quantum theory in explaining the specific heat of solids.

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- (c) Derive the expression for the Fermi-Dirac distribution function. Represent it graphically for $T = 0$ and $T \neq 0$.

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