

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:

3×20=60

(a) How are linear and angular momenta related to each other? Considering a system of mass points under the influence of forces derived from potentials only, prove that the generalised linear momentum is conserved. Establish further that the angular momentum of the above system is also conserved if the potential is centrally symmetric.

(b) Show that the mass–energy relationship in relativistic kinematics can lead to the equation

$$E^2 = c^2 p^2 + m_0^2 c^4$$

where E is the total energy of the particle of rest mass m_0 , linear momentum p and moving with a velocity v; c is the velocity of light in free space.

(c) Explain how Einstein's A and B coefficients are related to the phenomena of spontaneous and stimulated emission of radiation, respectively. Derive the relation between A and B. Establish that at very high frequency around X-ray wavelength regime, lasers cannot be made as easily as at low frequencies, e.g. far infra-red regime.

(d) Drawing a neat diagram, discuss how light travels through an optical fibre. Show that the numerical aperture of a commercially available optical fibre is around 0.25. Explain its physical significance.

2. (a) Considering the scattering of α -particles by the atomic nuclei, find out the Rutherford scattering cross-section. Explain the physical significance of the final expression. **40**

(b) What are constraints of motion? Explain with examples the holonomic and non-holonomic constraints. Discuss critically how can one overcome the limits of constraints by introducing generalised coordinates. **20**

3. (a) Show that the length L of an object moving with a velocity v is given in the direction of motion by

$$L = L_0 (1 - v^2/c^2)^{1/2},$$

where L_0 is the proper length and c is the velocity of light in free space. What will be the shape of a spherical ball while moving under relativistic regime? **20**

(b) For a transverse sinusoidal wave of wavelength λ propagating along negative x direction through a string fixed at a point, show that the nodes are located at $x=0, \lambda/2, \lambda, 3\lambda/2, \dots$ while the kinetic energy/unit length at the antinodes is given by

$$E = 2\rho A^2 \omega^2 \cos^2 \omega t \quad \text{20}$$

where, ρ , A and ω , are the mass density/unit length, amplitude of transverse displacement and angular frequency of the wave, respectively.

- (c) What do you understand by paraxial rays? Show that the effect of translation of a paraxial ray while travelling along a homogeneous medium is represented by a 2×2 matrix if the ray is initially defined by a 2×1 matrix. 20
4. (a) Why does one get three-dimensional image in holography? Explain with appropriate figures how can one construct and read a hologram. 20
- (b) What is the essential difference between interference and diffraction of light? How can you achieve Fraunhofer diffraction in the laboratory? Using the concept of Fraunhofer diffraction at a single slit, find out the intensity distribution produced by two slits of equal width. 20
- (c) Why does one see two image points for a single object point while viewed through a calcite crystal? What is this property of the crystal known as? What is an optic axis of a crystal? Explain the meanings of positive and negative crystals with one example for each kind. 20

SECTION 'B'

5. Answer any three of the following:

3×20=60

- (a) Prove the relation

$$\nabla^2(1/|\mathbf{r}-\mathbf{r}'|) = -4\pi\delta(\mathbf{r}-\mathbf{r}')$$

and hence show that

$$\phi(\mathbf{r}) = \int p(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'| d\mathbf{r}'$$

is a solution of the Poisson equation

$$\nabla^2\phi(\mathbf{r}) = -4\pi p(\mathbf{r}).$$

- (b) Show that the electric and magnetic field vectors, $\mathbf{E}$ and $\mathbf{B}$, in plane electromagnetic waves are mutually perpendicular in a plane normal to the direction of propagation. How are phases of $\mathbf{E}$ and $\mathbf{B}$ related to each other?
- (c) Discuss the differences in the assumptions underlying Einstein and Debye theories of specific heat C_v . Give schematic plots of C_v versus reduced temperature for these theories and elucidate the differences therein. Elaborate the meaning of the “law of corresponding states” for these plots.
- (d) Consider the expression

$$C_p - C_v = -T(\partial v/\partial T)_p^2(\partial p/\partial v)_T$$

and give reasoning regarding the values of T when $C_p = C_v$ for water. Also, evaluate $C_p - C_v$ for a vander Waals gas to elaborate that its value is larger for any real gas as compared to an ideal gas.

6. (a) Consider two long and straight current carrying wires placed parallel to each other a certain distance apart. Derive an expression for the force per unit length experienced by these wires. Discuss that the attractive (repulsive) nature of this force is related to the directions of flow of currents in the two wires. 30
- (b) Derive approximate expressions for the potential and the radial as well as the azimuthal components of the field due to an electric dipole at points far away from it. Also derive expression and hence describe the effect of a uniform electric field on a dipole which can rotate freely. 30
7. (a) Write down the macroscopic form of the Maxwell's equations in any isotropic (but inhomogeneous) medium and define the symbols appearing therein. Convert these equations in the integral forms to highlight the laws represented by these equations. 25

- (b) Describe physical significance of the displacement current considering the example of current flow through a capacitor. 15
- (c) Use the Planck formula for the blackbody radiation

$$u(\omega, T) = \frac{\hbar^2}{\pi^2 c^3} \omega^3 / (\exp(\beta \hbar \omega) - 1)$$
with $\beta = 1/k_B T$ to derive Wien's law, Rayleigh-Jeans law and Stefan-Boltzmann law. 20
8. (a) Starting from the expression $N = \sum_k \langle n_k \rangle$, where $\langle n_k \rangle$ is the average number of particles in the k^{th} quantum state, derive an expression for the average number of particles in the ground state of an ideal Bose gas. 15
- (b) Utilize the above expression to elaborate the concept of the Bose–Einstein condensation and discuss that the phenomenon explains qualitatively the properties in the low-temperature phase of liquid ^4He . 25
- (c) Show that the chemical potential of a system is an intensive quantity and is a function of temperature and pressure only. 20