

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:

20×3=60

(a) A particle moves in space with the Lagrangian

$$L = \frac{1}{2} m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V + A\dot{x} + B\dot{y} + C\dot{z}$$

where A, B, C are given functions of x, y, z. Find the corresponding Hamiltonian in terms of coordinates and momenta.

(b) A meson of rest mass π comes to rest and disintegrates to a muon of rest mass μ and a neutrino of zero rest mass. Show that the kinetic energy of motion of the muon is

$$T = ((\pi - \mu)^2 c^2) / 2\pi,$$

(c) Show that the ratio of the focal lengths of the two lenses in an achromatic doublet is given by

$$f_1/f_2 = -w_1/w_2$$

where w_1 and w_2 are the dispersive powers of the lenses of focal lengths f_1 and f_2 respectively.

(d) A siren of frequency 900 Hz is going towards a wall away from an observer at a speed 10 m/sec. Determine: (i) Frequency of sound directly heard from the siren. (ii) Frequency of sound reflected from the wall. (iii) Number of beats per second heard by the observer. (Velocity of sound = 330 m/sec)

2. (a) What do you mean by the moments and products of inertia? Show that the angular momentum vector is related to the angular velocity components by linear transformation relations.

20

(b) For a freely falling body from the height 'h' on the surface of the earth in the northern hemisphere with a latitude θ , show that the deviation of the body towards east at the final stage is given by $1/3 \omega \cos \theta (8h^3/g)^{1/2}$ [?], where ω is the angular velocity of the earth and 'g' is the acceleration due to gravity.

20

(c) If a single stage rocket fired vertically from rest at the earth's surface burns its fuel in a time of 30 sec and their relative velocity v_r [?] = 3 km sec^{-1} , what must be the mass ratio m_0/m for a final velocity v of 8 km sec^{-1} ?

20

3. (a) Write down the expression for the relativistic mass of a particle moving with a velocity v in terms of its rest mass? Establish from the above expression Einstein's mass-energy relation $E = mc^2$.

20

- (b) A thin converging lens and a thin diverging lens are placed coaxially at a distance of 5 cm. If the focal length of each lens is 10 cm, find for the combination (i) the focal length, (ii) the power, (iii) the positions of the principal points. 20
- (c) Write down the equation of motion for a damped harmonic oscillator assuming the damping force proportional to the velocity of the particle. Obtain the general solution for its displacement as a function of time. Discuss the cases of ‘overdamping’, ‘under-damping’ and ‘critical damping’. 20
4. (a) A quartz quarter wave plate is to be used with the sodium light ($\lambda = 5896 \text{ \AA}$). What should be its thickness? 20
- (b) Two microscope slides of length 10 cm each form a wedge. At one end they are in contact and at the other end they are separated by a thin wire of diameter d (see the diagram below). Interference fringes are obtained when illuminated vertically by a monochromatic light of $\lambda = 500$ nanometers. The fringe spacing is found to be 1.25 cm. Estimate the diameter of the wire.
[Figure: Air wedge: two glass slides touching at the left end and separated at the right end by a thin wire (circle) of diameter d , marked by a vertical double arrow d ; the length of the slides marked by a horizontal double arrow 10 cm.] 20
- (c) A narrow slit illuminated by monochromatic light of $\lambda = 6400 \text{ \AA}$ is placed at a distance of 3 metres from a straight edge. If the distance between the straight edge and the screen is 6 metres, calculate the distance between the first and the fourth dark bands. 20

SECTION 'B'

5. Answer any three of the following: 20×3=60
- (a) What is the volume density of the charge in a region of space, where the electrostatic potential is given by
- $$V = a - b(x^2 + y^2) - c \ln(x^2 + y^2),$$
- where a, b, c are constants?
- (b) A network PQRS is connected as shown in the figure below. Apply Kirchhoff's law and show that the current flowing through the 20Ω resistor PR is 0.029 A.
[Figure: Rectangle with corners S (top left), R (top right), Q (bottom right), P (bottom left). Side SR: 200Ω resistor. Side RQ: 60Ω resistor. Side QP: 4 V cell. Side PS: 5 V cell. Diagonal PR: 20Ω resistor. For both cells the longer (positive) plate is drawn on the side towards P.]
- (c) Show that the entropy of one gram molecule of an ideal gas is given by
- $$S = C_p \ln V + C_v \ln p + S_0$$
- (d) Given the temperature of the Sun's surface $T = 5755 \text{ K}$, radius of the Sun $= 6.9 \times 10^5$ [?] km, distance between the Earth and the Sun $= 1.5 \times 10^8 \text{ km}$. Estimate the solar constant (i.e. the energy received per sec per unit area of the Earth's surface). Assume $\sigma = 5.7 \times 10^{-8} \text{ W/m}^2\text{-K}^4$.
6. (a) A point charge q is held at a distance d in front of an infinite grounded conducting plane. What is the electric potential in front of the plane? 20
- (b) Find out the magnetic field inside a long solenoid carrying current i and having n turns per unit length. 20
- (c) Define the strength of a magnetic shell and calculate the magnetic potential at any neighbouring point due to this shell. Does the potential depend on the shape of the shell? 20
7. (a) What are the vector and scalar potentials? Derive Maxwell's wave equations in terms of these potentials. 20

(b) What do you understand by a perfect black body? Can it be realised in practice? Show that the ratio of the emissive power to the absorptive power for all bodies at a given temperature is equal to the emissive power of a perfectly black body. 20

(c) Show that the Poynting vector $\vec{S} = (\vec{E} \times \vec{H})$ represents the energy flow per unit area per unit time both in magnitude and direction in case of a plane electromagnetic wave. 20

8. (a) A gas expands isothermally from the pressure p_1 and volume V_1 to the pressure p_2 and the volume V_2 . Calculate (i) the change in internal energy. (ii) the change in entropy. (iii) the change in enthalpy. What will be the corresponding quantities when the gas expands adiabatically? 30

(b) Show that for a distribution of electrons, the number of electrons N_i in the energy state ϵ_i are given by

$$N_i = g_i / (A \exp(\epsilon_i / kT) + 1)$$

where g_i represents the number of quantum states in the energy level ϵ_i . Further state under what conditions this distribution law goes over to Maxwell-Boltzmann statistics. Show by drawing curves how the Fermi-Dirac distribution function varies with the energy at $T = 0$ and also at the other finite temperatures. 30