

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:
 - (a) Using the rocket equation and its integral find the final velocity of a single stage rocket. Given that (i) the velocity of the escaping gas is 2500 m/s, (ii) the rate of loss of mass is $[\frac{1}{2}m_0]$. (where m_0 is the initial mass and $0.27 m_0$ is the final mass). 20
 - (b) Two spaceships are moving at a velocity of $0.9 c$ relative to the Earth in opposite directions. What is the speed of one spaceship relative to the other? (c = velocity of light) 20
 - (c) A wave is represented by $\Psi_1 = 10\cos(5x + 25t)$. Find wavelength λ , velocity v , frequency f and the direction of propagation. If it interferes with another wave given by $\Psi_2 = 20\cos(5x + 25t + \pi/3)$, find the amplitude and the phase of the resultant wave. (All dimensions are in SI system). 20
 - (d) Derive the expression for resolving power of a diffraction grating with N lines. Calculate the minimum number of lines in the diffraction grating if it has to resolve the yellow lines of sodium (589.0 nm and 589.6 nm) in the first order. 20
2. (a) Using the Lagrangian for the system of a planet and the Sun obtain the equation of motion. Use them to get the equations for the orbit. 30
 - (b) Derive the relationship between the impact parameter and the scattering angle for the scattering of an α particle of charge $+2e$ by a nucleus of charge $+Ze$. Calculate the impact parameter for an angle of deflection of 30° if the kinetic energy of the alpha particle is 6×10^{-13} J. 30
3. (a) State Fermat's principle. Apply it to get the laws of reflection from a plane surface. 20
 - (b) The phase velocity of the surface wave in a liquid of surface tension T and density ρ is given by

$$v_p = \sqrt{(g\lambda/2\pi + 2\pi T/\lambda\rho)}$$
 Show that the group velocity v_g of the surface wave is given by

$$v_g = (g + (12\pi^2 T)/\rho\lambda^2) / (2\sqrt{((2\pi g/\lambda + 8\pi^3 T/\rho\lambda^3))})$$
20
 - (c) An observer A sees two events at the same space point ($\Delta x = \Delta y = \Delta z = 0$) and separated in time by $\Delta t = 10^{-6}$ s. Another observer B sees them to be separated by $\Delta t' = 3 \times 10^{-6}$ s. What is the separation in space of the two events as observed by B? What is the speed of B relative to A? 20
4. (a) How do you know that the light is a transverse wave? What is a quarter wave plate? How is it constructed? 20

- (b) Discuss the Fresnel diffraction pattern formed by a straight edge using the Cornu's spiral. 20
- (c) In an experiment using a Michelson interferometer, explain with the help of suitable ray diagrams (i) Why do we need extended source of light, (ii) Why do we get circular fringes, and (iii) Shifting of fringes inwards or outwards as we shift the movable mirror. 20

SECTION 'B'

5. Solve any three of the following:

- (a) Calculate the electric field for a point on the axis of a uniform ring of a charge 'q' and radius a. Show that the maximum value occur at $x = \pm a/2$. 20
- (b) Show that the potential energy of a charge Q uniformly distributed throughout the sphere of radius R is given by

$$PE = \frac{3}{5} \frac{Q^2}{4\pi\epsilon_0 R} \quad 20$$

- (c) Describe Carnot cycle and show that efficiency is given by

$$\eta = (Q_1 - Q_2)/Q_1 = (T_1 - T_2)/T_1$$

where the symbols have their usual meaning. 20

- (d) Derive the Bose-Einstein distribution for an ideal gas. 20

6. (a) Using Kirchoff's laws find currents in each branch of the circuit shown in the following diagram.

[Figure: Two-loop circuit with nodes a (top left), b (top middle), c (top right), d (bottom right), e (bottom middle), f (bottom left). Branch a-b: 12 Ω resistor. Branch b-c: 8 Ω resistor. Left branch f-a: 18 V cell, positive (longer) plate towards a (marked +), negative towards f. Middle branch b-e: 6 Ω resistor. Right branch c-d: 21 V cell, shorter plate towards c and longer plate towards d. Bottom wire f-e plain; between e and d a 3 Ω resistor (upper) in parallel with a 6 Ω resistor (lower).]

20

- (b) A Geiger tube consists of a wire of radius 0.2 mm and length 12 cm and a co-axial metallic cylinder of radius 1.5 cm and length 12 cm. Find (i) the capacitance of the system, and (ii) the charge per unit length of the wire when the potential difference between the wire and the cylinder is 1.2 kV. (Assume the dielectric constant of the gas in the tube to be 1) 20

- (c) A series LCR circuit with $L = 2$ H, $C = 2$ μ F and $R = 20$ Ω is powered by a source of 100 volts and variable frequency. Find (i) the resonance frequency, f_o , (ii) the value of Q (iii) the width of resonance Δf and (iv) the maximum current at resonance. 20

7. (a) Why did Maxwell have to introduce the idea of displacement current? Derive the wave equation from Maxwell's laws. Obtain Fresnel's formula for reflection and transmission coefficients of the electric vector when it is perpendicular to the plane of incidence. 30

- (b) What are vector and scalar potentials for the electromagnetic field? Are they unique? Explain what are Coulomb's and Lorentz gauges. Derive the electromagnetic wave equation in Lorentz gauge and show that it is equivalent to Maxwell's equation. 30

8. (a) Discuss the phenomenon of Bose-Einstein condensation. Obtain the expression for the condensation temperature. Briefly comment on observation of Bose-Einstein condensate. 30

- (b) A bulb filament is constructed from a tungsten wire of length 2 cm and diameter 50 μ m. It is enclosed in a vacuum bulb. What temperature does it reach when it is operated at a power of 1 watt? Given: (i) Emissivity of tungsten $\epsilon = 0.4$ (ii) Stefan's constant $\sigma = 5.67 \times 10^{-8}$ watt/m²K⁴. 30