

PHYSICS

(Paper I)

Time Allowed : Three Hours
Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

(Please read each of the following instructions carefully before attempting questions)

There are EIGHT questions divided in two Sections.

Candidate has to attempt FIVE questions in all.

Question Nos. 1 and 5 are compulsory and out of the remaining, any THREE are to be attempted choosing at least ONE question from each Section.

All questions carry equal marks.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Word limit in questions, wherever specified, should be adhered to.

SECTION 'A'

1. Answer any three of the following:

- (a) The two atoms in a molecule of a gas interact according to the potential

$$\phi(r) = A/r^6 + B/r^{12}$$

[?] r being the separation distance between the two atoms. Determine A and B if the potential energy $\phi(r) = \phi(r_0)$ at the equilibrium separation $r = r_0$.

- (b) A smooth sphere A of mass m and speed u impinges obliquely on a sphere B of mass M ($M > m$) at rest. The velocity of A before collision makes an angle θ with the line of centres at the time of impact. Show that A is deflected through a right angle if

$$\tan^2 \theta = e(M-m)/(M+m)$$

e is the coefficient of restitution.

- (c) A series LCR circuit has $L = 20$ mH, $C = 0.5$ μ F and $R = 10$ Ω . The circuit is driven by an alternating emf with amplitude 200 V. Calculate (i) resonance frequency (ii) the current at resonance frequency (iii) Q value and (iv) half width of the resonance current.
- (d) A thin quartz plate for incident light of $\lambda = 6100$ $\text{\AA}$ produces, a phase difference $(45/4) \lambda$ between the ordinary and extraordinary rays. What will be the nature of polarisation of the emergent light when plane polarized light of wavelength 4500 $\text{\AA}$ is incident on it.

2. (a) The molecules of a gas are made up of four non coplanar atoms. Enumerate the translational, vibrational and rotational degrees of freedom of each molecule. Hence obtain the specific heat at constant volume C_v of the gas. Does the value so obtained agree with the observed value in general? If not why?

- (b) Prove the latent heat equation

$$(\delta L / \delta T)_{sv} - L/T = C_s - C_p$$

where C_s and C_p are specific heats of saturated vapour and the liquid in contact with it respectively. Given the following values referring to 1 gm of water at 100 $^\circ$ C: $L = 539$ Cal/gm, $(\delta L / \delta T)_{sv} = -0.64$ Cal K^{-1} gm $^{-1}$, $C_p = 1.01$ Cal K^{-1} g $^{-1}$. Calculate C_s . Explain why the specific heat takes a negative value.

- (c) Distinguish between spatial and temporal coherence of light. What kind of coherence is required for interference and diffraction? Explain why it is possible to produce interference effects using two tuning forks of the same frequency but not by two independent sources of light of the same wavelength.

3. (a) Define dispersive and resolving powers of a plane transmission grating and obtain expressions for the two. Show that the first and second order spectra produced by such a grating will never overlap when the incident light contains wave-lengths in the range 4000 Å to 7000 Å.
- (b) Write down the expression for the energy distribution for the black body radiations at temperature T . Show that this expression goes into the Rayleigh-Jeans distribution at one end of the frequency spectrum and the Wiens distribution at the other end.
- (c) A steel ball 1.00 mm in diameter at constant speed of 0.176 cm s^{-1} in a large vessel filled with oil. Calculate the dynamic viscosity of oil. Given density of steel = 7700 kg m^{-3} and that of oil = 900 kg m^{-3} . Deduce the mathematical relation needed for the calculation.
4. (a) Consider a spiral spring of length L and mass M suspended vertically from a rigid support. A mass m is attached to the lower end of the spring. The mass m is now pulled down through a small distance and is then released. If M and m are comparable describe the motion. What elastic constant of the material of the spiral spring provides the restoring force?
- (b) Show that the angular momentum of a particle located at position $\mathbf{r}$ relative to the origin of co-ordinates is given by $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ where $\mathbf{p}$ is the linear momentum of the particle. Using this result prove that the angular momentum of a system of particles can be expressed as the sum of their angular momentum around the centre of mass and the angular momentum around the origin of a single particle of mass equal to the total mass of the system located at the centre of mass.
- (c) Calculate the minimum plate separation in a Fabry-Perot interferometer to obtain free spectral range of 0.05 Å in the wavelength region 5000 Å . Calculate also the smallest resolvable wavelength difference for reflectivity of 0.95.

SECTION 'B'

5. Answer any three of the following:
- (a) A block A of mass 6 kg is placed on a plane inclined to the horizontal at an angle 30° . It is joined to another block B of mass 18 kg by means of a massless string going round a pulley. B hangs vertically. The pulley can be approximated as a uniform circular disc of mass 2 kg and radius 10 cm. The string from A to the pulley is parallel to the plane. The kinetic friction coefficient is $1/3\sqrt{3}$. Calculate the acceleration of B and the tensions in the two segments of the string.
- (b) Explain the terms resolving power and magnifying power of an optical instrument. On what parameter do these physical quantities depend in case of a telescope? For a given re-solving power what is the optimum magnifying power in this case?
- (c) Consider a resonant cavity of volume V of a container to which a neck of area of cross section a and length l is attached. Show that the product of the square of resonance frequency n and volume V is constant. Comment on the correction needed to the derived relation between n^2 and V . How will you determine this correction?
- (d) A μ meson travels towards the earth's surface from high up in atmosphere with a speed of $0.99 c$. It decays after travelling a distance of 6 km. In what time does the μ meson decay as measured by observers in reference frames (i) bound to the earth and (ii) bound to the meson itself.
6. (a) Prove that $\int_A^B \delta Q/T$ evaluated along a reversible path joining the states A and B does not depend on the path chosen. Hence define the entropy function S . Calculate the entropy change in an ideal gas undergoing a state change from (V, P) to $(2V, P/2)$ for three suitably chosen different paths and show that the result turns out to be the same all the cases.

- (b) Prove that the path of particle moving in a central force field is a plane curve and that the angular momentum of the particle remains constant in time.
- (c) State Huygens principle and use it to establish Snell's law of refraction of light. How does this treatment of refraction differ from that given on the basis of Newton's corpuscular theory?
7. (a) State the basic assumptions of Debye theory of specific heat of solids and write the expression for C_v derived from this theory. Show that this expression yields the famous T^3 law of specific heat at low temperature. Discuss the extent to which the theory agrees (or disagrees) with observation on specific heat through the variation of the Debye characteristic temperature Θ_D with temperature, in general.
- (b) What was the basis for light to be accepted as an electromagnetic wave? The $\mathbf{E}$ vector in a light wave polarised in the (x, y) plane is expressed as
- $$\mathbf{E}(x,y,z,t) = \mathbf{E}_0 \hat{e} \sin[\omega t - k(x+y)]$$
- Determine the propagation and the polarisation vectors.
- (c) How is an electron volt connected to other units of energy like the Joule or the erg? Determine the speed of an electron of energy 1.3 Mev assuming its rest energy to be 0.5 Mev.
8. Write short notes on any three of the following:
- (a) Gyroscope
- (b) Brownian motion
- (c) Ruby laser
- (d) Clausius Clapeyron equation
- (e) Ultrasonic
- (f) Coupled oscillations